Redhill A-Level Maths

Workbook AS Applied

STATISTICS

Statistical sampling:
A **population** is a group of items, people, dogs, cats, etc that you wish to find information about.
A **sample** is part of the population.
A **parameter** is a numerical characteristic of a population such as mean or standard deviation. It is fixed value as all the data items of the population are used.
A **statistic** is a quantity calculated from a sample such as the mean or standard deviation. ***Since these are taken from part of the population they usually vary from sample to sample.***
A sample is used to make inferences about the population.

Suppose we study the population of dogs in Brighton. A parameter of this population would be the mean height of all dogs in the city. A statistic would be the mean height of 50 of these dogs.

Types of data:
Categorical or qualitative data - this is data that can be put into categories and is described by using words
e.g. colour (red, blue, green, etc), months (Jan, Feb, March, etc), types of music etc

Quantitative data - this is numerical data that can be:
Discrete i.e. can only take particular values e.g. any quantity that can be counted, the number of chairs in a room, the number of people at a football match, shoe sizes.
Continuous i.e. can take any numerical value e.g. any quantity that can be measured, length, height, weight, temperature.

Primary and secondary data:
Data that **you** collect and analyse from surveys, questionnaires, observations or experiment is called primary data. Data that you obtain from someone else (books of tables, data banks on computer, or on line) is called secondary data.

Sampling:
If every member of a population is observed or measured, this is called taking a census. It gives completely accurate results. Ideal if planning for schools or housing requirements etc, but it is costly and time consuming. It could be impractical. e.g. If you wanted to test the life of a battery it would be ludicrous to test every battery - you would not have any batteries left to sell!

A sample is used to make inferences about the population.

Simple random sampling:
This is where **every member of the population being sampled has an equal chance of being selected.**
Consider the situation where you want to find out about leisure activities of students in a particular school. You might take a sample of say 50 students and use this to make inferences about the whole population of students in that school.
A method to obtain a random sample of 50 students from the whole population of students in that school, would be to number all the students and then use the random number generator on a calculator/computer, or use random number tables to select the 50 students to answer your questions.

1. Give one advantage and one disadvantage of simple random sampling.

Systematic sampling:
This is a sample that is obtained by taking the required data at regular intervals from an ordered list. Normally the start point from the list would be randomly selected. e.g Taking every 10^{th} student from an alphabetic list of all students. You could start on say the 7^{th} student, and then the 17^{th}, 27^{th} etc.

2. Give one advantage and one disadvantage of systematic sampling.

Stratified sampling:
This is where you want to make sure that different groups (strata), that are of different sizes within the population to be sampled, are represented fairly in the sample. The strata are usually sex, age, religion, etc. For stratified sampling the population is divided into groups (strata). The proportion of each group in the population should be the same proportion in the sample. Within each strata a simple random sample is selected.

Considering the situation above there may be more girls than boys in the school. So if there are 300 girls and 200 boys in the school, you should take your sample to reflect this.

$\frac{300}{500} \times 50 = 30$ girls and $\frac{200}{500} \times 50 = 20$ boys

You would then use simple random sampling to select the 30 girls and the 20 boys.
You could use other groups e.g. each year group.

3. Give one advantage and one disadvantage of stratified sampling.

Cluster sampling:
In Custer sampling the whole population is divided into groups or clusters. Then simple random sampling is used to choose appropriate clusters. The whole cluster, or a simple random sample within the cluster, is used to collect the data. Often used where geographical areas are being sampled.
Example: A survey of farmers is to be taken in England to determine the amount of pesticides used in their fields. The researcher may divide country into clusters of farms by county and then simple random sampling method is applied to choose the clusters to be surveyed.

4. Give one advantage and one disadvantage of cluster sampling.

This may seem similar to stratified sampling. A major difference between cluster and stratified sampling is that in cluster sampling a cluster is perceived as a sampling unit, whereas in stratified sampling only specific elements of strata are accepted as sampling unit.

Quota sampling:
A sampling method of gathering representative data from a group. As opposed to random sampling, quota sampling requires that representative individuals are chosen out of a specific subgroup with no random element as to the subjects chosen.
For example, a researcher might ask for a sample of 100 females between the ages of 20-30. The interviewer would then just collect data from any 100 females between the ages of 20-30.

5. Give one advantage and one disadvantage of quota sampling.

Opportunity or convenient sampling:
This is where subjects are selected because of their convenient accessibility and/or proximity to the researcher.

6. Give one advantage and one disadvantage of opportunity sampling.

7. Say whether the following types of data are Categorical (Qualitative), Discrete or Continuous.
   (a) The make of a car. (b) The number of people in the car. (c) The speed of the car.
   (d) The time for the car to travel a journey. (e) The number of parts that make up the engine.

8. There are 120 year 8, 152 year 9 and 128 year 10 students in a particular school. A survey of 100 of these students is to be undertaken. If a stratified sample is to be used, how many of each year group should be used in the survey?

In any question you may be asked to give reasons why a particular sampling method may be biased, or not very practical. You will need to look at the question to decide yourself, but you may look at things like:

- Every member of the population has an equal chance of being selected.
- Exclude part of the population.
- Some chosen element may not be present. e.g. Seats in a theatre may be empty when taking a sample of theatre goers.
- Sensitive data - some respondents may not disclose private information to an interviewer - Could write a response and deposit in a box, or complete online to keep their responses anonymous.
- Where the sample is taken. e.g. Sampling people leaving a sports hall, when you are looking into how the population of a town rate the leisure facilities in that town. (Biased as all respondents use some facilities.)
- Time to collect the data and whether the respondents would wish to be included.

etc.

Measures of central tendency and variation:

Mode: This is the value that occurs most often.

Median: This is the middle value once the data have been arranged in order. If there are two numbers in the middle, add them up and divide by two.

Mean: Add up all the values and divide by the number of values. $\bar{x} = \frac{\sum x}{n}$

Range: Range = The highest value – The lowest value

Interquartile Range: Interquartile Range = Upper Quartile – Lower Quartile

These are measures of dispersion. They give a numerical value for the spread of the data.

If the data set is small you can usually work out the Quartiles by inspection.

For 60 data items the median is the mean of the 30^{th} and 31^{st} values.

The lower quartile is then the middle of the lower half of values i.e. the mean of the 15^{th} and 16^{th} values.

The upper quartile is then the middle of the upper half of values i.e. the mean of the 45^{th} and 46^{th} values.

1^{st} 2^{nd} ……14^{th} 15^{th} 16^{th} 17^{th} ……29^{th} 30^{th} 31^{st} 32^{nd} ……44^{th} 45^{th} 46^{th} 47^{th} ……59^{th} 60^{th}

↑ Lower Quartile ↑ Median ↑ Upper Quartile

For large n, use ¼ n , ½ n and ¾ n to find the lower quartile, median and upper quartile respectively.

Note: If there are any outliers, the range would be adversely affected and give a larger value for the range. This value would not be representative of the spread of the data. The interquartile range is used as it usually will not include any outliers and so it gives a value that is more representative of the spread of the data.

Variance and Standard deviation are measures of dispersion.
They give a numerical value for the spread of the data. The smaller the value, the smaller the spread of the data. This enables you to compare different data sets.
Notation:
Population mean and standard deviation are denoted by μ and σ respectively.
Whereas the mean and standard deviation from a sample are denoted by $\bar{x}$ and s respectively.
If the question is not specific about what type of data is being used, just use $\bar{x}$ and s.

Edexcel students.

Mean,

$$\bar{x} = \frac{\sum x}{n}$$

$$S_{xx} = \sum (x-\bar{x})^2$$

$$= \sum x^2 - \frac{(\sum x)^2}{n}$$

Standard deviation,

$$s = \sqrt{\frac{S_{xx}}{n}}$$

For a population this is sometimes called the *root mean square deviation from the mean.*

OCR students.

Mean,

$$\bar{x} = \frac{\sum x}{n}$$

Standard deviation,

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

AQA students.

Population mean,

$$\mu = \frac{\sum x}{n}$$

Population variance,

$$\sigma^2 = \frac{\sum (x-\bar{x})^2}{n}$$

$$= \frac{\sum x^2}{n} - \bar{x}^2$$

Standard deviation,

$$\sigma = \sqrt{variance}$$

An **unbiased estimate** of the population mean equals the sample mean.
Sample mean,

$$\bar{x} = \frac{\sum x}{n}$$

An **unbiased estimate** of the population variance/standard deviation from a sample is given by:

Variance,

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

$$= \frac{1}{n-1}\left\{\sum x^2 - \frac{(\sum x)^2}{n}\right\}$$

Standard deviation,

$$s = \sqrt{variance}$$

OCR MEI students.

Mean of a set of data,

$$\bar{x} = \frac{\sum x}{n}$$

$$S_{xx} = \sum (x-\bar{x})^2$$

$$= \sum x^2 - \frac{(\sum x)^2}{n}$$

$$= \sum x^2 - n\bar{x}^2$$

Variance, $s^2 = \frac{S_{xx}}{n-1}$

Standard deviation,

$$s = \sqrt{variance}$$

9. For the following set of data find the mean and standard deviation.
   3, 5, 7, 9 Give your answers to 3 significant figures.

Could just use your calculator.
To get a greater understanding and to make it easier to remember, it is best to show working and check answer using calculator.

10. For a particular population of 20 numbers: $\sum x = 184$ and $\sum x^2 = 2648$
    Find the mean and standard deviation. Give your answers to 3 significant figures.

Sometimes you are given questions in this form. So you need to be able to use the particular formulae that includes $\sum x$ and $\sum x^2$

11. The heights, h, of 9 plants were found to have a mean of 12 cm and a standard deviation of 2. (i) Find $\sum h$ and $\sum h^2$ (ii) An additional plant has a height of 2 cm. Find the mean and standard deviation of all 10 plants. Give your answers to 3 significant figures.	*Don't forget to square the s.d. to get the variance.* *Use the formulae to work out $\sum h$ and $\sum h^2$ Then add in the new number and the new number squared respectively. Then use the formulae to work out the new mean and s.d.*

Using your calculator - *When entering data values into your calculator make sure that you know which symbols give the required statistic (there are various similar symbols e.g. σ n and σ n–1).*
Enter the following as data items/list and hence find the mean and standard deviation, either direct from your calculator or by obtaining Σx, Σx^2 etc. and using the formula.
Do not just write down two numbers. Write, for example, mean or $\bar{x}$ = and standard deviation or s =
If you incorrectly round a number you may lose all marks having shown no working. You should write down your calculator value, to say six significant figures, and then round to 3 significant figures, or how many significant figures, or decimal places you are told.

12. Find the mean and standard deviation of the following data: (Give your answers to 3 significant figures.)
(a) 5.6, 6.9, 8.5, 6.1

(b) 24, 35, 67, 25, 36, 62, 21, 13, 0, 54

(c) 568, 691, 999, 541, 853

Edexcel students should know about coding. You should know that:
If you have used the coding $y = \frac{x-b}{a}$ to work out a mean and/or standard deviation you need to work backwards to find the mean and/or standard deviation of the original data using:
$\bar{x} = a\bar{y} + b$ and/or $s_x = a\, s_y$

13. For Edexcel students.
A set of 50 data values is known. The coding $y = \frac{x-100}{3}$ is used to give $\sum y = 65$ and $\sum y^2 = 3625$ Find the mean and standard deviation of the data.

Outliers:

These are the values that are far from the mean. **These values should be examined** to decide whether to discard them from the data set, or keep them. They may be from a mistake in collecting the data, or a written/data entry, or a misread, or a respondent giving a false response, or inaccurate, etc, or genuine data. You should be able to comment on the likely effect of removing an outlier.
Rules for identifying outliers should be specified, but generally you can use the following when commenting on outliers in a data set:

- More than 1.5 × (Interquartile Range) from the nearer quartile. When IQR is known.
- More than 2 × (Standard Deviation) away from the mean. When Mean and St dev are known. Edexcel students would have used - More than 3 × (Standard Deviation) away from the mean.

Working with frequency tables:

The marks obtained by 50 students in a test are summarised below:

Mark	0	1	2	3	4	5	6	7	8	9	10
No. of students	1	2	2	3	6	9	12	7	5	2	1

The mode: this is the value that occurs most often.
The mode is 6 marks. Careful: A common mistake is to put 12 for the mode.

The median:
This is the middle value once the data have been arranged in order. If there are two numbers in the middle add them up and divide by two.

Here the marks are already in order.
Two halves of 25, so the median is given by the 25th and 26th students. (or the ½ (n + 1)th student)
Count up to see which mark the 25th and 26th students obtained.
Here they both have a score of 6, so the median is 6 marks.

The mean, variance and standard deviation:
Using the previous formulae for your exam board (detailed on page 6) with
$\sum f$ for n , $\sum fx$ for $\sum x$, $\sum fx^2$ for $\sum x^2$ and $\sum f(x-\bar{x})^2$ for $\sum(x-\bar{x})^2$

14. Complete the table and find the mean and standard deviation of marks:

mark, x	numbers of students, f	fx	x^2	fx^2
0	1	$0\times1=0$	$0^2=0$	$0\times1=0$
1	2	$1\times2=2$	$1^2=1$	$1\times2=2$
2	2	$2\times2=4$	$2^2=4$	$4\times2=8$
3	3			
4	6	$4\times6=$		
5	9		$5^2=$	
6	12			$36\times12=$
7	7			
8	5			
9	2			
10	1			

$\sum f=$ $\sum fx=$ $\sum fx^2=$

Enter the following as data items/lists and hence find the mean and standard deviation, either direct from your calculator, or by obtaining Σx, Σx^2 etc. and using the formula.

15. Find the mean and standard deviation (to 3 sig. fig.) for each of the following frequency tables:

(a)

Score	5	6	7	8	9	10	11	12
Frequency	3	5	8	15	11	6	4	1

(b)

Goals	0	1	2	3	4
Frequency	10	21	12	5	1

(c)

Points	0	1	2	3	4	5	6	7	8
Frequency	8	5	2	4	7	6	5	1	2

16. For a particular set of numbers $\sum f = 29$, $\sum fx = 116$ and $\sum fx^2 = 498$
Find the mean and standard deviation (to 3 sig. fig.) of the set of numbers.

17. For the following frequency table, find the mean and standard deviation of marks (to 3 sig. fig.):

mark	frequency			
9	5			
10	9			
11	12			
12	6			
13	2			
Totals				

Mean, variance and standard deviation from grouped data:

Here we assume that the members in each group have a mean that is the middle of the group.
You can then use the formulae for a frequency table.

18. Complete the table and find an estimate for the mean and standard deviation of the height of plants:

Height of plants (cm)	Frequency f	Midpoint of group, x	fx	x^2	fx^2
5-14	10	9.5	$9.5 \times 10 = 95$	$9.5^2 = 90.25$	$90.25 \times 10 = 902.5$
15-19	12	17	$17 \times 12 =$	$17^2 =$	
20-24	20				
25-29	18				
30-34	12				
35-39	6				
40-44	4				
45-54	4				

$\sum f =$ $\sum fx =$ $\sum fx^2 =$

Estimates to 3 sig.fig.

Be careful when working out the midpoint of a group (e.g. for the above group 5-14 should it be 9.5 or 10?)
To find the midpoint of a group add the upper and lower boundaries and divide by 2.
For the above groups: 5 - 14 midpoint $(4.5 + 14.5) \div 2 = 9.5$, 15 - 19 midpoint $(14.5 + 19.5) \div 2 = 17$ etc.

19. Estimate the mean and standard deviation for this distribution (to 3 sig. fig.).

Length (cm)	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59
Frequency	6	12	40	17	5

Write down the mid class values.
Then try using your calculator for this one.

20. Phil collects data on the heights of 40 students (all those taking Maths) and records them as shown:

Height in m, x	1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90
Frequency , f	2	4	5	8	10	7	3	0	1

Given $\sum f = 40 \quad \sum fx = 66.95 \quad \sum fx^2 = 112.3575$

Calculate the mean and standard deviation of the height (to 3 sig. fig.).

Median and other percentiles from grouped data - Interpolation:

This is easiest to follow by looking at an example. Just learn the method.

21. Complete this example: From the distribution of the heights of plants below, use interpolation to find the median and quartiles. Find also the 40th percentile.

Height (cm)	Frequency	Cumulative frequency
20-29	3	
30-39	14	
40-49	25	
50-59	30	
60-69	18	
70-79	9	
80-89	1	

Cumulative frequency is just a running total.
3
3+14
17+25
etc

Complete the cumulative frequency column. (Not absolutely necessary - but useful.) n large.

Median at 50th value

$$\text{Median, } Q_2 = \frac{8}{30} \times 10 + 49.5 = 52.2\text{cm (to 1 d.p.)}$$

n = 100 n large - so use - ***median*** *at (½ n)th value.*
(This is the 8th value in the 50 - 59 group i.e. 50 – 42 = 8
Width of group 49.5-59.5 is 10
Frequency of group is 30
So this fraction $\frac{8}{30}$ of the way into the group added to the starting value of the group, which is 49.5)

Complete:

lower quartile at $\quad^{th}$ value

lower quartile, Q_1 =

upper quartile at $\quad^{th}$ value

upper quartile, Q_3 =

Lower quartile:(¼ n)th value.
Work out using above method.
Upper quartile:
(¾ n)th value.

The 40th percentile is the value 40% of the way through the distribution.

40th percentile at $\quad^{th}$ value

40th percentile =

n = 100 so the 40th percentile is the 40th value (40/100 of 100)
The 40th value is the 23rd value in the group 40-49

This method assumes that values are evenly spread through an interval. It enables you to calculate an **estimate** for the median (and other percentiles) without drawing a cumulative frequency graph.

The percentile is a statistic. It explains the relative position of an object in a group of objects.
The percentile is used to compare the value with that of the other scores in the group, or to know what value is in a certain position within the scores.
The difference between two percentiles is called the interpercentile range.

22. In a survey, a random sample of 110 teenagers were asked how many minutes, to the nearest minute, they spent on their mobile phone on a particular day. The results are summarised in the table below. Use linear interpolation to estimate the median and interquartile range to 3 significant figures.

Number of minutes	Frequency
0-5	21
6-7	17
8-10	19
11-15	26
16-25	16
26-50	11

Data presentation and interpretation:
You should know your work at GCSE level on bar graphs, stem and leaf plots, box plots, histograms, etc. Some of which are recapped here.

Frequency diagrams:

Bar graphs, vertical line graphs and histograms are all types of frequency diagrams.
Bar graph or vertical line graph for categorical, or discrete data and a histogram for continuous data.

Making comments about diagrammatic representations:
When having to make comments you should think about (where applicable.)

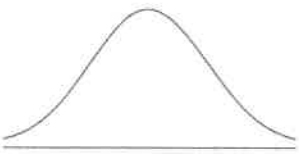
- outliers
- population density (population per km^2)
- weather disruption affecting data (very hot or very cold for a period)
- catastrophic event affecting data (sales/income/deliveries/etc)
- rounding of values. (A 0 in a table may be rounded from 0.35 - totals in a table may not be the sum of the individual numbers as 3.4 plus 5.3 = 8.7 rounded to 9, but 3.4 and 5.3 rounded gives 3 plus 5 = 8)

etc.

When comparing two diagrams you should consider: the median/mean/mode; range; interquartile range frequency; shape of the distribution; percentages from median and quartiles.
If these values are easily read/calculated then they should be quoted.

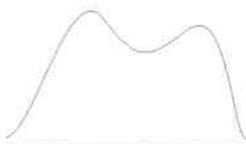
Describing the shape of a distribution:

From a diagram of the distribution of values (histogram, bar chart, frequency polygon, ……) we get an idea of the overall 'shape' of the distribution and describe the different types as follows:

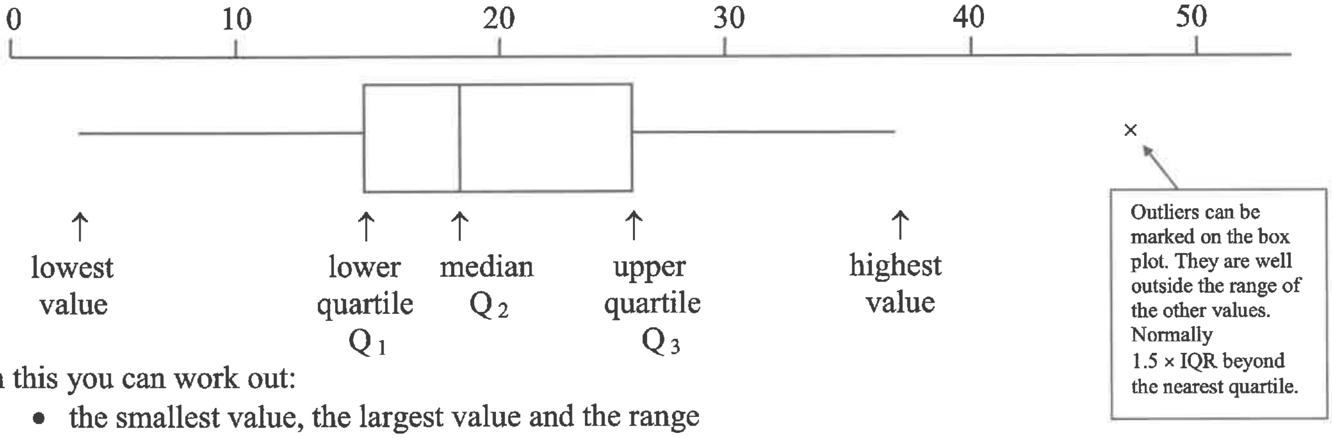

symmetrical


positive skew


negative skew


bimodal

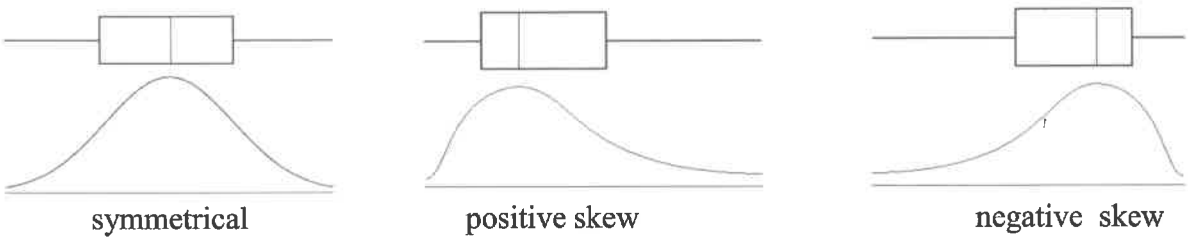
Box plots: Also called 'box and whisker plots'.
This diagrammatic representation of data shows the location and spread of the data.
Scale:



From this you can work out:

- the smallest value, the largest value and the range
- the median and the upper and lower quartiles
- the spread of the middle 50% , the interquartile range (IQR)
- the 'shape' of the distribution.

Also more than one box plot can be drawn using the same scale, so that different data sets can be compared.

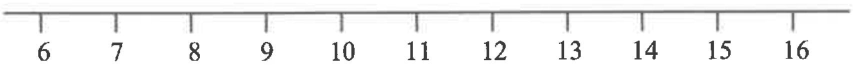


Note: The median divides the area of the histogram into two equal halves.

23. Twenty people were asked to estimate the length of a line, to the nearest cm, drawn on paper. Their replies were as follows: 9, 13, 12, 7, 11, 12, 11, 11, 13, 10, 13, 8, 12, 12,10, 12, 13, 15, 6, 12
Draw a box plot and comment on the shape of the distribution.

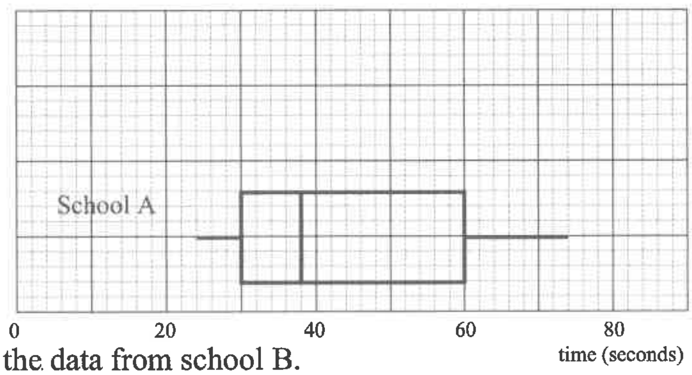
Put the data in order.
Find the median and quartiles (write them down!)
Draw the box plot.
Comment.

Median Q_2 = Q_1 = Q_3 =



24. The time taken, to the nearest minute, for some students to complete a task in school A, is summarised in the boxplot below.
(a) Write down the time by which 25% of the children in school A had completed the task. State the name given to this value.

For school B the least time taken by any of the children was 20 seconds and the longest time was 76 seconds. The three quartiles were 34, 46 and 60 respectively.



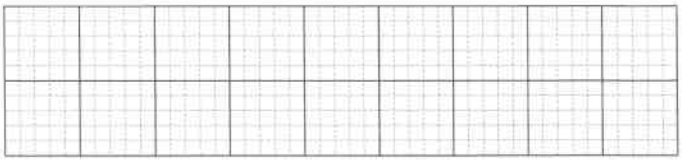
(b) On the same diagram draw a box plot to represent the data from school B.
(c) Compare and contrast these two box plots. (Give 4 comments.)

25. The waiting times for 20 patients at a surgery were recorded to the nearest minute.
7, 5, 16, 12, 17, 8, 5, 11, 6, 8, 7, 14, 26, 16, 10, 7, 15, 8, 11, 18

(a) Find the median and interquartile range of the waiting times.

An outlier is an observation that falls either 1.5 × (interquartile range) above the upper quartile, or 1.5 × (interquartile range) below the lower quartile.

(b) Draw a boxplot to represent these data, clearly indicating any outliers.
(c) Comment on the skewness of these data.



Working with grouped data - Histograms:
You should have met these on your GCSE course so just a short reminder.
When you draw a histogram:

- First establish the boundaries for each interval.
  e.g. for height of plants that have been measured to the nearest cm, in the interval 20-24, the lower boundary is 19.5 and the upper boundary is 24.5
  Do not use the class boundaries for your horizontal axis. Use a continuous horizontal axis going up in 1s or 2s or 5s or 10s etc. Draw the blocks, as accurately as you can, from the lower boundary to the upper boundary. You would normally use graph paper.
- Find each interval width. width = upper boundary – lower boundary
- For a histogram

the area represents the frequency

The area is proportional to frequency.

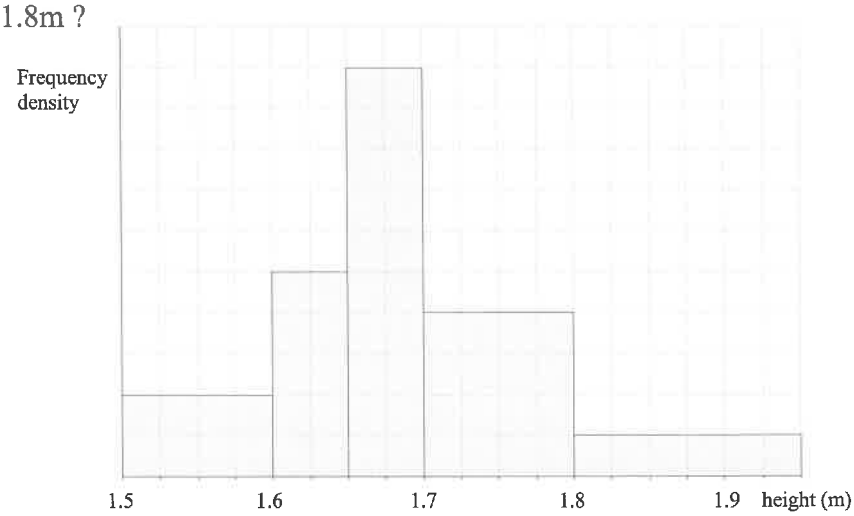
So when the constant of proportionality is 1 (the easiest form to consider)

frequency = interval width × height of block

therefore

$$\text{height of block} = \text{frequency density} = \frac{\text{Frequency}}{\text{interval width}}$$

Interpreting histograms:
Sometimes the area equals the frequency, as in the examples above. Alternatively the area is proportional to the frequency and it is easiest to work out how much the area of 1 square (the larger the better) is worth and complete the question from this knowledge. Best by example: This histogram shows the distribution of heights of 240 male students in a college.
How many male students are of height 1.7 - 1.8m ?



Total area represents 240 students.

Total number of squares = 60

Therefore each square represents
$240 \div 60 = 4$ students

Squares in 1.7 - 1.8 group = 16
Students of height 1.7 - 1.8m = $16 \times 4 = 64$

26. In a survey, a random sample of 104 teenagers were asked how many minutes, to the nearest minute, they spent on their mobile on a particular day. The results are summarised in the table below.
A histogram was drawn and the group (8 - 10) minutes was represented by a rectangle that was 1.5 cm wide and 3 cm high.
Calculate the width and height of the rectangle representing the group (16 - 25) minutes.

Number of minutes	Frequency
0-5	20
6-7	16
8-10	18
11-15	25
16-25	15
26-50	10

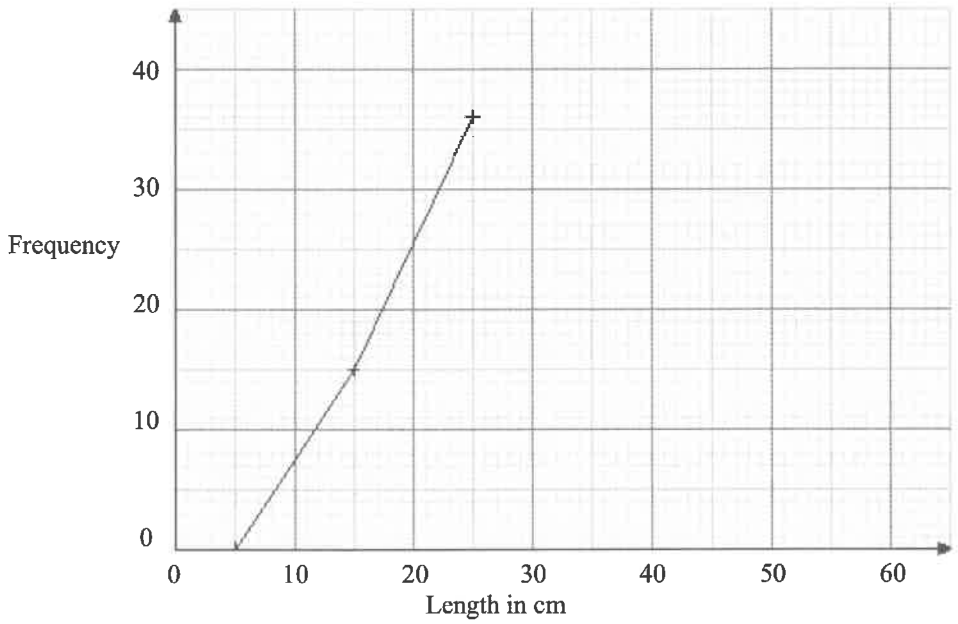
Frequency polygons:

Here you plot the mid point of each group on a graph and join them up with straight lines.
The advantage over histograms is that you can draw more than one frequency polygon on a graph. This enables you to compare different sets of grouped data with the same total frequency.

27. Complete this example. This table gives the distribution of the length of fish in a lake (taken from a sample).

Length, l (cm)	$0 \le l < 10$	$10 \le l < 20$	$20 \le l < 30$	$30 \le l < 40$	$40 \le l < 50$	$50 \le l < 60$
Frequency	0	15	36	32	17	0

Draw a frequency polygon to represent this distribution.



The middle of the first group $0 \le l < 10$ is 5. So plot a point at 5 for the length and 0 frequency.

The middle of the second group is 15. So plot a point at 15 for the length and 15 for the frequency. Join this point to the first point.

Repeat for the other frequencies.

A second sample of 100 fish is taken one month later and the distribution is shown below.

Length, l (cm)	$0 \le l < 10$	$10 \le l < 20$	$20 \le l < 30$	$30 \le l < 40$	$40 \le l < 50$	$50 \le l < 60$
Frequency	0	6	28	40	24	2

Draw on the same grid the frequency polygon for the second sample.
Make two comparisons between the 1st and 2nd sample.

(1) (2)

Modal group:
When data values are grouped the modal group is that with the largest frequency density. When all the groups have the same width the group with the greatest frequency is the modal group.

If two values or two groups occur more frequently than the others, and the stem and leaf diagram, or histogram, has a "double hump" appearance, the distribution is said to be BI-MODAL, even if one "hump" is taller than the other.

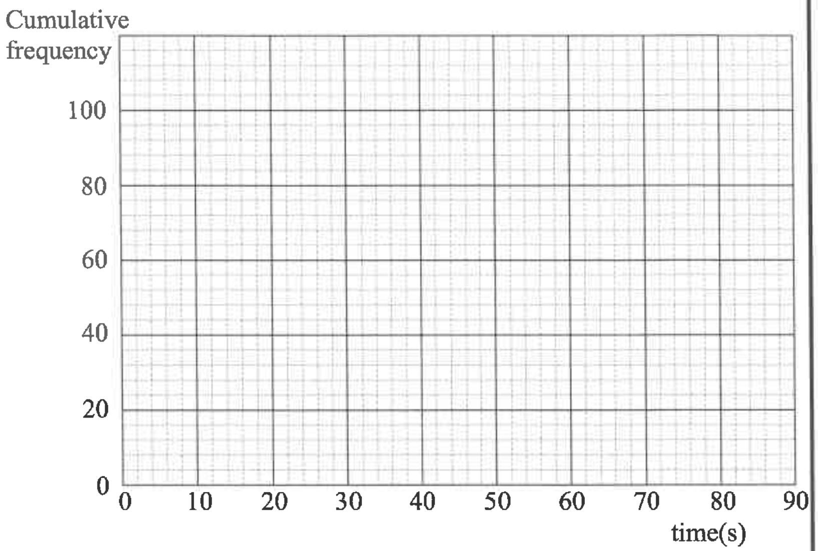
Cumulative frequency:

Note: If you need to find the median and quartiles use interpolation. Only draw a cumulative frequency curve if the question asks you to do so.

Again, you have met this at GCSE so an example should remind you what to do.

28. The table shows the distribution of times taken by 100 people to complete a task.

Time (sec)	Frequency	Cumulative frequency
20-29	3	
30-39	14	
40-49	25	
50-59	30	
60-69	18	
70-79	9	
80-89	1	



Complete the cumulative frequency column.

Draw the cumulative frequency graph.

Find the median and quartiles.

Median Q_2 =

Lower quartile, Q_1 =

Upper quartile, Q_3 =

Inter quartile range =

Use graph paper for cumulative frequency in exams.

Use a continuous horizontal axis going up in 1s or 2s or 5s or 10s (It will be easier to read off values when finding the median etc)

Remember to plot top of the group against cumulative frequency.
i.e. 29.5 and 3
39.5 and 17
49.5 and 42
etc.

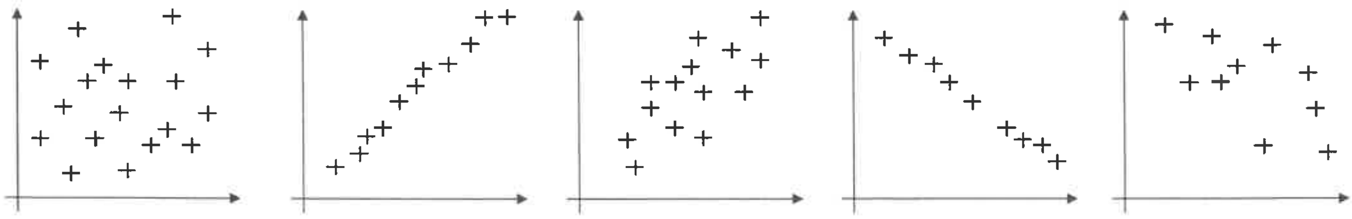
Plot your points as accurately as you can.

Draw the lines on the graph. Be as accurate as possible and read off the values as accurately as you can. This will be easier to do on graph paper.

Scatter diagrams and regression lines for bivariate data:

Correlation and scatter diagrams.

Correlation is the degree to which two or more attributes, or measurements, on the same group of elements show a tendency to vary together.



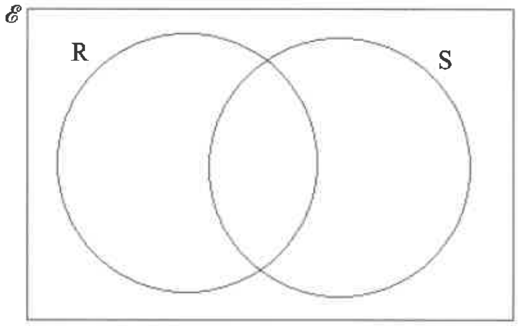
No correlation | Strong positive correlation | Weak positive correlation | Strong negative correlation | Weak negative correlation

If there is a correlation between two sets of data, it means they may be connected in some way.
You must always be careful with the linear correlation. Just because two variables indicate good correlation it does not automatically mean they are related (associated) - it may be due to another variable, or just coincidence. You need to have a reason to suspect that they are connected (or not).

You can use Venn diagrams to show the actual members/elements in each section.

32. Put in these set members on the Venn diagram.

$\mathscr{E} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$
R = $\{2,4,6,8,10,12,14,16,18,20\}$
S = $\{3,6,9,12,15,18\}$



If an element of the set ε is chosen at random (a single trial) write down:

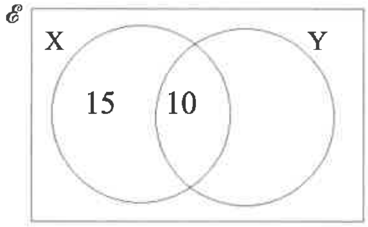
(i) P (the number is even) =

(ii) P (the number is a multiple of two <u>OR</u> three) =

(iii) P (the number is not a multiple of six) =

You can also use a Venn diagram to show the number of elements in each section.

$n(\mathscr{E}) = 80$
$n(X) = 25$
$n(Y) = 36$
$n(X \cap Y) = 10$ Complete the diagram.



33. If an element from ε is chosen at random (a single trial) find:

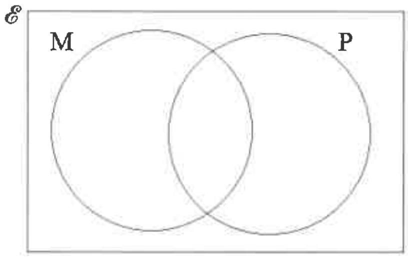
(i) P (it is in $X' \cap Y$)

(ii) P (it is in $X' \cap Y'$)

Conditional probability:

You can use Venn diagrams to help you understand 'conditional probability'. Consider this example:

In a college there are 36 mathematics students and 28 physics students. 10 students study both subjects. Complete the Venn diagram,



A student is picked at random.

Given that the student studies physics, what is the probability that he also studies maths?
i.e. the student studies physics, so we know that he is one of the 28 students-of these 28 students, 10 also study maths (the students in $P \cap M$). You can see that we have a 10 out of 28 chance of choosing a maths student given that he studies physics.

Mathematically $P(M \mid P) = \dfrac{\textit{The number in the intersection of M and P}}{\textit{The number in P}}$

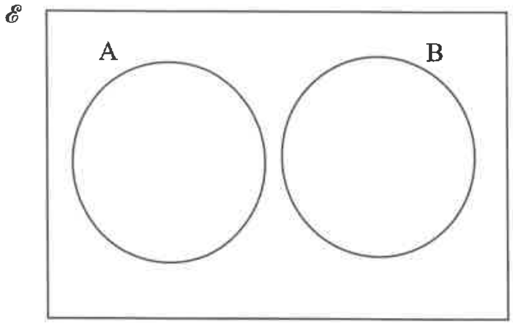
For P ($M \mid P$) we say "the probability of *M*, given *P* has occurred".

$$= \frac{n(P \cap M)}{n(P)}$$

$$= \frac{10}{28}$$

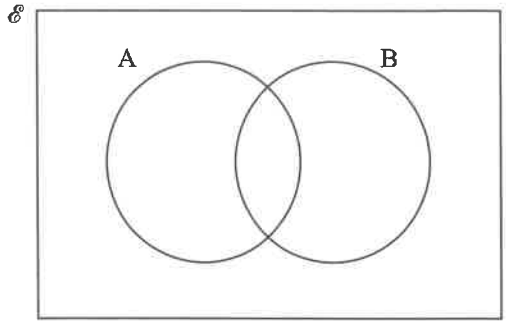
Addition rule:

Probability of one event <u>or</u> another event happening.



Mutually exclusive events.
(Events that cannot happen simultaneously)
e.g. P(Picking an ace or a king from a pack of cards)

$$P(A \cup B) = P(A) + P(B)$$



Not mutually exclusive events.
e.g. P(Picking an ace or a club from a pack of cards)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

34. In a group of 30 students, all study either History, or Geography, or both. 20 study History and 21 study Geography. If a student is picked at random what is the probability that he or she studies both subjects?	Draw a Venn diagram.
35. The probability that a fish in a fish tank has spots is 0.5 and the probability that it has stripes is 0.4. If the probability that the fish has spots and stripes is 0.2, find the probability that a fish, chosen at random, has spots or stripes.	Put the probabilities on a Venn diagram.
36. A number is picked at random from the numbers 1 to 20 inclusive. Given that the number is even, find the probability that it is a multiple of 5.	Conditional probability. Draw a Venn diagram with two sets. Even and multiples of 5.

Independent events: (Multiplication rule.)

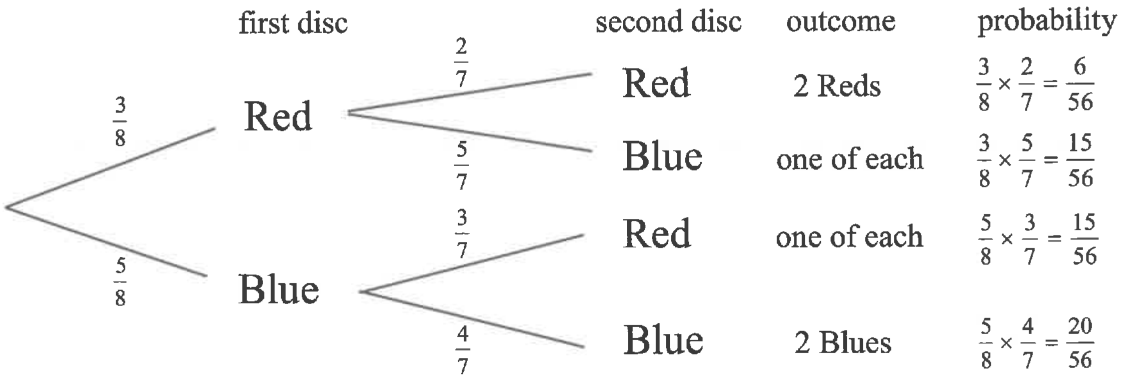
Here the occurrence of one has no effect on the other.
e.g. Drawing a king then a queen from a pack of cards if the first one drawn is replaced.

Independent events $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$

If you are asked to show that events are independent you must work out the probabilities: $P(A)$, $P(B)$ and $P(A \cap B)$ and then workout $P(A) \times P(B)$ to see if is the same as $P(A \cap B)$. If it is you can then say "therefore A and B are independent".

Tree diagrams: These can help you to understand a probability question.

Example: There are three red discs and 5 blue discs in a bag. A disc is chosen without looking, its colour noted and then a second disc is chosen.
Find the probability that the discs are (i) both red (ii) different colours.



$P(2 \text{ reds}) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

Probability of a red <u>and</u> a red ('<u>and</u>' so multiply).

$P(\text{one of each colour}) = \left(\frac{3}{8} \times \frac{5}{7}\right) + \left(\frac{5}{8} \times \frac{3}{7}\right) = \frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}$

Probability of red <u>and</u> blue **or** blue <u>and</u> red.
('<u>and</u>' so multiply …'**or**' so add)

We can also understand **conditional probability** from looking at a tree diagram. (Here we are looking at two trials)

$P(\text{Red} \cap \text{Blue}) = P(\text{Red}) \times P(\text{Blue given Red has occurred})$

$P(R \cap B) = P(R) \times P(B \mid R)$

or rearranging $P(B \mid R) = \dfrac{P(R \cap B)}{P(R)}$ as we had before.

Dependent events.

For $P(B \mid R)$ we say "the probability of B given R has occurred".

Tables and probability:
You may be asked to calculate probabilities from a table.

37. The result of a survey of how students travelled to college are given in the following table.

A student is selected at random from this group.
Find the probability that the selected student is
(i) a year 12 student
(ii) a year 13 student that walks to college
(iii) a student that cycles given that they are in year 13
If W and C denotes the events that a student walks and cycles respectively and T is the event that the student is in year 12, show that the event $W \cup C$ is independent of the event T.

	Year12	Year 13
Walk	58	64
Cycle	14	8
Other	28	28

A mix of questions:

38. Find the probability of obtaining at least one six in a throw of 3 dice.	*P(1 six) = 1 – P(no sixes) So need to find P(no six and no six and no six) i.e. the multiplication rule.*
39. In a game for 2 players, a turn consists of throwing a die once or twice. If a player throws less than 6 on the first throw, the score is the score on the die. If a player throws a 6 on the first throw, they throw again and their score is the total score of the two throws. Find the probabilities of (i) scoring more than 9 in a turn (ii) scoring a total of more than 20 in two successive turns (iii) obtaining equal scores in two successive turns.	*List the pairs that give a score of more than 9* *For (ii) again list the possibilities and write down the probability of getting the scores. Work out the probability.* *A table of probabilities of each score will help for (iii)*
40. Given that $P(X) = 0.8$, $P(Y) = 0.5$ and $P(X \cup Y) = 0.9$ find $P(X \cap Y)$	*Use the formulae for $P(A \cup B)$ Draw a Venn diagram if it helps to see what you are trying to find.*
41. If X and Y are independent events such that $P(X) = 0.25$, $P(Y) = 0.15$ find (i) $P(X \cap Y)$ (ii) $P(X \cup Y)$	Check what independent means!

42. Six identical balls numbered 1 to 6 are placed in a bag. If two balls are chosen one after the other find the probability of obtaining a total score of 10 or more if
(a) the first ball is replaced (b) the first ball is not replaced.
Two balls are chosen one after the other
(c) Find the most probable score in case (a) and (b) giving their probabilities.

A tree diagram for each case may make your work messy. A list of possible outcomes may be better.

DISCRETE RANDOM VARIABLES

Discrete - can only take particular values.
Random - cannot determine the outcome for any trial.
Variable - can take different numerical values.

The discrete probability distribution:

If a discrete random variable X can take the values $x_1, x_2, x_3, \ldots\ldots, x_n$ with probabilities $p_1, p_2, p_3, \ldots, p_n$ respectively, the **probability distribution** is written:

x	x_1	x_2	x_3	....	x_n
$P(X=x)$	p_1	p_2	p_3		p_n

These are the values that the random variable can be.

These are the corresponding probabilities.

The probability that the random variable equals x

and $$\sum_{i=1}^{i=n} p_i = 1$$ i.e. The probabilities add to give 1

43. Complete the probability distribution for the number of heads obtained when three coins are thrown.

The random variable is:
H = 'the number of heads obtained when three coins are thrown'

h	0	1	2	3
$P(H=h)$	$\frac{1}{8}$			$\frac{1}{8}$

Don't forget the 0.
List the possible outcomes if you are not sure.

44. There are 10 discs in a bag numbered 1,2,2,3,3,3,4,4,4 and 4. If S is the random variable 'the score on the disc when one is chosen at random', write down the probability distribution of S.

The probability distribution is just the table of possible scores and their probabilities.

45. A random variable X has probability function f(x) given by

$$f(x) = \frac{3x+1}{50} \quad ; \quad x = 1, 2, 3, 4, 5$$

Write down the probability distribution of S.

The discrete uniform distribution:
If you have a discrete random variable where the outcomes are all 'equally likely' to turn up, then you have a discrete uniform distribution. The probabilities are the same in the probability distribution.

The cumulative distribution function:

$$F(x_0) = \sum_{x \le x_0} P(X = x)$$

Just the sum of the probabilities up to x_0

46. The cumulative distribution function for a discrete random variable, X, is shown below.

x	1	2	3	4	5
F(x)	0.4	0.6	0.7	0.9	1

Work out (a) $P(X < 3) =$

(b) $P(X \le 4) =$

(c) $P(X = 2) =$

(d) The probability distribution.

x	1	2	3	4	5
P(X=x)					

(a) Does not include 3. Answer straight from the table.

(b) Straight from the table. F(4)

(c) F(2) – F(1)

THE BINOMIAL DISTRIBUTION

Consider the number of sixes obtained when a fair die is thrown five times.
Note:

* We are interested in only the two outcomes: throw a six (Success) or not throw a six (Failure).
* The probability of obtaining a six is not affected by previous throws (the trials are independent).
* There are a set number of throws (trials) i.e. five.

If S = obtain a six from a throw and F = do not obtain a six from a throw

P (no sixes from 5 throws) i.e. F F F F F is given by $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{5}{6}\right)^5$

P (one six from 5 throws) i.e. S F F F F is given by $\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4$

or F S F F F is given by $\frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4$

or F F S F F is given by $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4$

or F F F S F is given by $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} = \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4$

or F F F F S is given by $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4$

$$P(\text{one six}) = {}^5C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4 \quad \text{or} \quad \binom{5}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4$$

The number of ways of choosing one place for the S from five places is: ${}^5C_1$ or $\binom{5}{1}$

So there are: ${}^5C_1 = 5$ lots of $\left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4$

P (two sixes from 5 throws) i.e. S S F F F is given by $\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

or S F S F F is given by $\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

and so on.

$$P(\text{two sixes from 5 throws}) = {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 \quad \text{or} \quad \binom{5}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$

The number of ways of choosing two places for the S from five places is: ${}^5C_2$ or $\binom{5}{2}$

So there are: ${}^5C_2 = \frac{5.4}{2.1} = 10$ lots of $\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

Notice that although we have found the probabilities for throwing a die 5 times, these probabilities could have been for any binomial situation with 5 trials (and is easily extended for more or less trials) where the probability of success is $^1/_6$ (and easily changed for other probabilities).

47. Complete: P(three sixes from 5 throws) =

P(four sixes from 5 throws) =

P(five sixes from 5 throws) =

IN GENERAL.
Use the binomial distribution when:

- There are set number of trials.
- You are only interested in two outcomes SUCCESS and FAILURE.
- The probability of Success is the same for each trial (INDEPENDENT trials).

If X is binomially distributed where X = "the number of successes"
then

$$X \sim B(n, p)$$

where n = the number of trials
p = the probability of success

The mean or expected number of successes $= n \times p$

and the probability of r successes,

$$P(X = r) = {}^nC_r\, p^r q^{n-r} \quad \text{or} \quad \binom{n}{r} p^r q^{n-r}$$

where $q = 1 - p$

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

48. Example: One in five shirts produced in a factory are rejected. Find the probability that if six shirts are chosen at random:
(a) 1 is rejected
(b) 3 are rejected
(c) at least 3 are rejected

X = the number of rejected shirts in a sample of 5

Write this down.

$$X \sim B\left(6, \frac{1}{5}\right)$$

This means X follows a binomial distribution where the number of trials is 6 and the probability of success is 1/5

Write this down. $n = 6$, $p = 1/5$

(a) $$P(X = 1) = {}^6C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^5$$

Show how you get the answer.

This means - the probability that the number of rejected shirts equals 1

$$= 6 \times \frac{1}{5} \times \frac{1024}{3125} = \frac{6144}{15625} = 0.393216$$

Show working.

(b) $$P(X = 3) = {}^6C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^3$$

$$= \frac{6.5.4}{3.2.1} \times \frac{1}{125} \times \frac{64}{125} = 0.08192$$

(c) $$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - [\,P(X = 0) + P(X = 1) + P(X = 2)\,]$$

P(at least 3) means P(3 or more) which is the same as 1 – P(less than 3) [Or can be worked out as P(X=3) + P(X=4) + P(X=5) + P(X=6)]

Complete:

$P(X = 0) =$

Do not forget this one!

$P(X = 1) =$

Show working.

$P(X = 2) =$

$P(X \geq 3) = 1 - [\quad + \quad + \quad]$

Work out the answer.

Note: The expected (or mean) number of shirts rejected $= 6 \times {}^1/_5 = 1.2$

<table>
<tr><td>
49. In a multiple choice test there are 10 questions, each with 4 possible answers. If a candidate guesses the answers at random, find the probability that, out of 10, he scores:
(a) 0
(b) 1
(c) less than 3
(d) at least 5
Complete:

X =

X ~

(a) P(X =   ) =

(b) P(X =   ) =

(c) P(X <   ) = P(X =   ) +

(d) P(X ≥   ) = 1 − P(   )
</td><td>
In words.

The distribution. n = 10 p = ¼

Write out the formula with the numbers you know.
Work it out. Show your working.

Write out the formula with the numbers you know.
Work it out. Show your working.

Add up the probabilities less than 3.
You will have to work out P(X=2) the others you worked out in (a) and (b).

Easiest to work out the probabilities that X = 0,1,2,3,4 and subtract from 1. As you have already worked some out!
Longer method would be to work out the probabilities that X = 5,6,7,8,9,10 and add them up.

Note: the expected (or mean) score = 10 × ¼ = 2.5
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50. A three sided spinner, numbered 1 to 3, is spun eight times. If the numbers are equally likely to turn up, find the probability of obtaining:
(a) six 2s
(b) more than five 2s
</td><td>
n =
p =

Write X = ...

Write X ~ .....

(a) Write P(X =........

(b) Write P(X >........
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Cumulative Binomial Probability Tables:

Binomial Probabilities can be found using tables or your calculator. Learn how to use them!

Remember: The tables give *cumulative* probabilities. So using the extract below for a random variable X where:

$X \sim B(20, 1/6)$ i.e. $n = 20$ and $p = 1/6$

$P(X = 0) = 0.0261$
$P(X \le 1) = P(X = 0) + P(X = 1) = 0.1304$
$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.3287$

If you wish to find $P(X = 2)$ you must work out $P(X \le 2) - P(X \le 1)$

i.e. $P(X = 2) = P(X \le 2) - P(X \le 1)$
$= 0.3287 - 0.1304$
$= 0.1983$

n	x \ p	....	1/6	0.200	0.250	0.300	1/3	....	2/3	0.700	0.750
20	0	....	0.0261	0.0115	0.0032	0.0008	0.0003	....			
	1	....	0.1304	0.0692	0.0243	0.0076	0.0033	....			
	2	....	0.3287	0.2061	0.0913	0.0355	0.0176	....			
	3	....	0.5665	0.4114	0.2252	0.1071	0.0604	....			
	4		0.7687	0.6296	0.4148	0.2375	0.1515	....	0.0000		
	5		0.8982	0.8042	0.6172	0.4164	0.2972	....	0.0002	0.0000	
	6		0.9629	0.9133	0.7858	0.6080	0.4793	....	0.0009	0.0003	0.0000
	7		0.9887	0.9679	0.8982	0.7723	0.6615	....	0.0037	0.0013	0.0002
	8		0.9972	0.9900	0.9591	0.8867	0.8095	....	0.0130	0.0051	0.0009
	9		0.9994	0.9974	0.9861	0.9520	0.9081	....	0.0376	0.0171	0.0039
	10		0.9999	0.9994	0.9961	0.9829	0.9624	....	0.0919	0.0480	0.0139
	11		1.0000	0.9999	0.9991	0.9949	0.9870	....	0.1905	0.1133	0.0409
	12			1.0000	0.9998	0.9987	0.9963		0.3385	0.2277	0.1018
	13				1.0000	0.9997	0.9991		0.5207	0.3920	0.2142
	14					1.0000	0.9998		0.7028	0.5836	0.3828
	15						1.0000		0.8485	0.7625	0.5852
	16								0.9396	0.8929	0.7748
	17								0.9824	0.9645	0.9087
	18								0.9967	0.9924	0.9757
	19								0.9997	0.9992	0.9968
	20								1.0000	1.0000	1.0000

51. Complete: For $X \sim B(20, 1/6)$

$P(X \le 6) =$ $P(X \le 7) =$

Straight from the tables or calculator.

$P(X = 7) =$

$P(X = 4) =$

Remember you need to subtract.

$P(X < 3) =$

Careful - here it is < not ≤
So $P(X < 3) = P(X \le 2)$

$P(X > 8) =$

Careful - here it is > not ≤
So $P(X > 8) = 1 - P(X \le 8)$

52. For $Y \sim B(20, 0.7)$

$P(Y \le 10) =$ $P(Y \le 17) =$

Straight from the tables.

$P(Y = 12) =$

$P(Y = 18) =$

Remember you need to subtract.

$P(Y < 16) =$

Careful - here it is < not ≤
So $P(Y < 16) = P(Y \le 15)$

$P(Y > 16) =$

Careful - here it is > not ≤
So $P(Y > 16) = 1 - P(Y \le 16)$

$P(Y \ge 14) =$

Careful - here it is ≥ not ≤
So $P(Y \ge 14) = 1 - P(Y \le 13)$

53. A packet of sweets contains red and/or yellow sweets. In the factory twice as many red sweets are mixed with the yellow sweets, before a machine randomly puts them into packets of twenty sweets. (a) Find the probability that a packet contains at least 15 red sweets. The packets are then put into boxes that hold five packets. (b) Find the probability that a box contains at least 3 packets, containing at least 15 red sweets?	
(a) $X =$	*In words - the number of……*
$X \sim$	*The distribution*
$P(X \geq 15) =$	$= 1 - P(X \leq 14)$
$=$	*From tables or calculator.*
$=$	*The answer.*
(b) $Y =$ $Y \sim$ $P(Y \geq \quad) =$	*Careful! (b) is a different Binomial situation. Success is a packet having at least 15 red sweets. Failure is not. The number of trials (packets) is 5. The probability of success is the answer to part (a)* *Use 1 – again.*

Binomial distribution questions.

54. If $X \sim B(8, 0.4)$, find (i) $P(X \leq 2)$ (ii) $P(X = 4)$

55. If $Z \sim B(6, 0.225)$, find (i) $P(Z = 5)$ (ii) $P(Z > 4)$

56. If $Y \sim B(10, ¼)$, find (i) $P(Y \leq 3)$ (ii) $P(Y = 3)$

57. If $X \sim B(6, 3/8)$, find (i) $P(X < 2)$ (ii) $P(X \leq 2)$

58. The probability a darts player hits the bull (when aiming for it) is 0.7 Assuming independence, what is the probability that he gets at least six bulls from nine throws?

59. If $X \sim B(n, 0.4)$ and $P(X \leq 5) = 0.7535$, find n.

60. Six coins are thrown into the air. Assuming the coins are unbiased, what is the probability of obtaining four or more heads?

61. Five fair dice are thrown in a game. What is the probability of obtaining at least three sixes?

62. The probability a student is late for a lesson is 0.1 For a period covering 10 lessons, find the probability that he is late: (i) for exactly 3 lessons (ii) for less than 3 lessons.

63. A bag contains 4 red balls and 6 blue balls. When a ball is drawn, without looking, its colour is noted and the ball is replaced. If eight balls are drawn, find the probability that:
(i) exactly 4 are red (ii) less than three are red (iii) more than 6 are red.

64. In a factory producing light bulbs, the probability that one is faulty is 0.15
A batch of 20 light bulbs is tested. Find the probability that:
(i) less than 2 are faulty (ii) 5 are faulty (iii) less than 5 are faulty
(iv) more than 5 are faulty (v) at least 5 are faulty.

Hypothesis testing:

Example:

Consider the number of heads obtained when a coin is thrown fifteen times. How many times would a head have to turn up in order to indicate a bias towards heads at the 5% level?

X = 'the number of heads obtained' $X \sim B(15, \frac{1}{2})$

H_0: $p = \frac{1}{2}$ (the coin is fair)
H_1: $p > \frac{1}{2}$ (the coin is biased towards heads)

Null hypothesis and the alternative hypothesis.

5% significance level one tailed test

i.e. 5%

From tables or calculator:

$P(X \geq 11) = 1 - P(X \leq 10) = 1 - 0.9408 = 0.0592$ (i.e. > 0.05) The probability that the number of heads is greater than 10

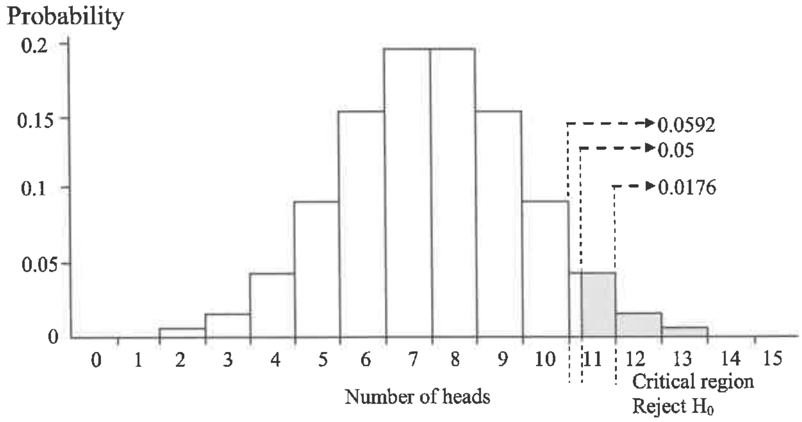
$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9824 = 0.0176$ (i.e. < 0.05) The probability that the number of heads is greater than 11

Placing the probabilities on a sketch of the distribution of probabilities enables you to see that 5% (i.e. 0.05) is somewhere in the '11 bar'.

The diagram shows the distribution of probabilities with the critical region.

12 is the critical value (i.e. the first value wholly in the critical region).

12, 13, 14, 15 all lie wholly in the critical region.



We conclude that, at the 5% level, the coin is biased towards heads if we obtain 12 or more heads.

65. The probability that patients have to wait more than 15 minutes in a health practice is 0.3 One of the doctors claims that there is a decrease in the number of patients that have to wait more than 15 minutes. He records the waiting time for the next 20 patients and finds that 3 wait more than 15 minutes. Is there evidence at the 5% level to support the doctor?

Complete:

H_0:

H_1:

Write in the null and alternative hypotheses.

X = the number of patients that wait more than 15 minutes

$X \sim B($, $)$ 5% significance level one tailed test

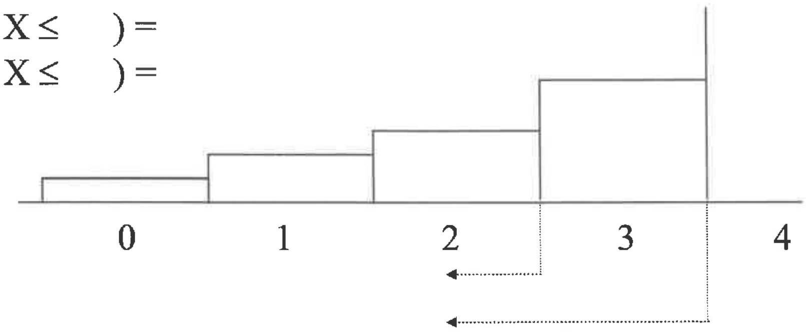
State the distribution.

$P(X \leq \quad) =$

$P(X \leq \quad) =$

Use tables or calculator to find the probability of 3 or less waiting more than 15 mins. This is the p-value.

Use tables or calculator to find P(X≤2) to see where the critical region actually is (not needed but aids understanding).



Write in the probabilities and approximately where the 5% line is. Shade the critical region.

Comment:

Accept or reject H_0.

Note: The *p*-value is defined as the probability, under the null hypothesis, of obtaining a result equal to or more extreme than what was actually observed.

66. The probability that James wins a game of chess against his computer is 0.4 He says that he has improved his game. In the next 8 games he wins 6. Test James' claim at the 5% level.

Complete:

H_0:

H_1:

Write in the null and alternative hypotheses.

X =

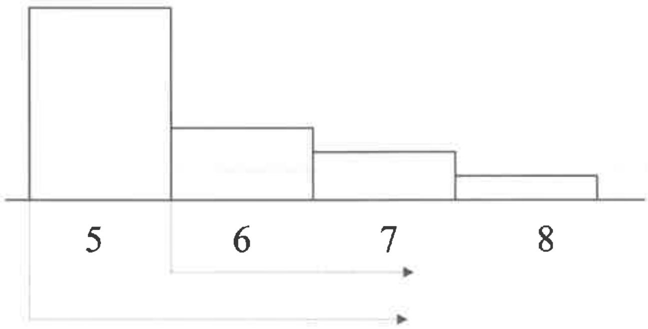
X ~ B(,) 5% significance level one tailed test

State the distribution.

P(X ≥) = 1 – P(X ≤) =

Use tables or calculator to find the probability of <u>6 or more</u> wins. (Remember this is 1 – P(X≤5). This is the p-value.

Use tables or calculator to find out P(X≥5) to see where the critical region actually is. (Not needed but aids understanding.)



Write in the probabilities and approximately where the 5% line is. Shade the critical region.

Comment:

Accept or reject H_0.

67. In Bestco stores 60% of their customers that buy a TV set will buy their own brand. The area manager says that in a particular store they sell less than expected. In a sample of 12 customers that bought a TV from that store, 5 bought the Bestco brand. Does this support his claim at the 10% level? What is the critical value?

Note this is 10%

Complete:

H_0:

H_1:

Write in the null and alternative hypotheses.

X =

X ~ B(,) 10% significance level one tailed test

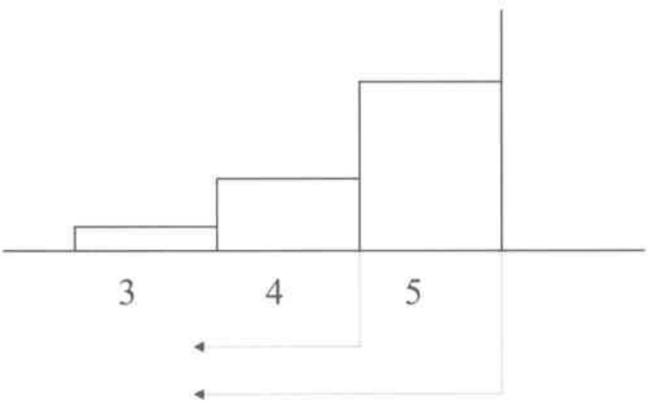
State the distribution.

P(X ≤) =

P(X ≤) =

Use tables or calculator to find the probability of <u>5 or less</u> customers buying Bestco. This is the p-value.

Use tables or calculator to find out P(X≤4) to see where the critical region actually is. (Not needed but aids understanding.)



Write in the probabilities and approximately where the 10% line is. Shade the critical region.

Comment:

Accept or reject H_0.

Critical value:

Two tailed test:

Use this when the change could be < or >. If it is not clearly stated in the question whether you are testing that the probability is less than, or more than the Null Hypothesis, you should be testing for a change - you would use a two tailed test.

A 10% significance level gives a critical region of 5% at each tail.

68. In a constituency, normally 50% of the residents turn up to vote in the local elections. To test whether there was a change in the attitude of residents, a random sample of 20 constituents were asked if they were going to vote in forthcoming elections. 15 said they would. Does this represent a change at the 5% level? What values lie in the critical region?

Complete: H_0:

H_1:

Write in the null and alternative hypotheses.

X =

$X \sim B(\quad,\quad)$ 5% significance level

two tailed test $\therefore$ 2.5% each end of distribution.

State the distribution.

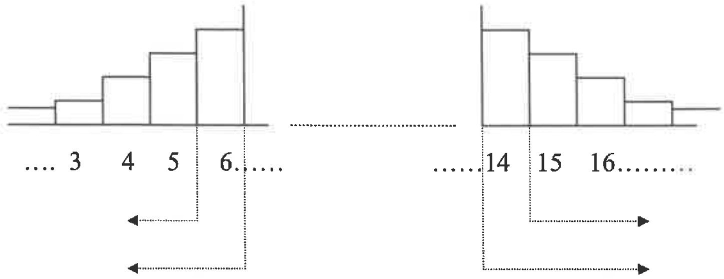
$P(X \geq \quad) = 1 - P(X \leq \quad) =$

Use tables or calculator to find the probability of 15 or more voting.

$P(X \geq \quad) = 1 - P(X \leq \quad) =$

$P(X \leq \quad) =$ $\qquad P(X \leq \quad) =$

Use tables to find out where both ends of the critical region actually are. (Needed for the last part.)



Write in the probabilities and approximately where the 2.5% line is at each end of the distribution. Shade the critical region.

Comment:

Accept or reject H_0.

Critical region: { }

69. A packing company has mixed sweets delivered with 70% red and the rest blue. The company wishes to test whether there is a change in the ratio of red to blue sweets. To do this they take a sample of 20 sweets of which 10 are red. Does this indicate a change at the 8% significance level? What numbers of red sweets would indicate a change at this level?

Mixed questions 1

70. Describe the main features and uses of a box plot.

71. The estimated speeds of a sample of 30 vehicles travelling along a stretch of road are shown below.

1	4
2	4 6 6 8 8
3	0 2 4 4 6 8 8 8
4	0 0 2 2 2 4 8
5	0 2 6 6 8
6	4
7	0 4
8	2

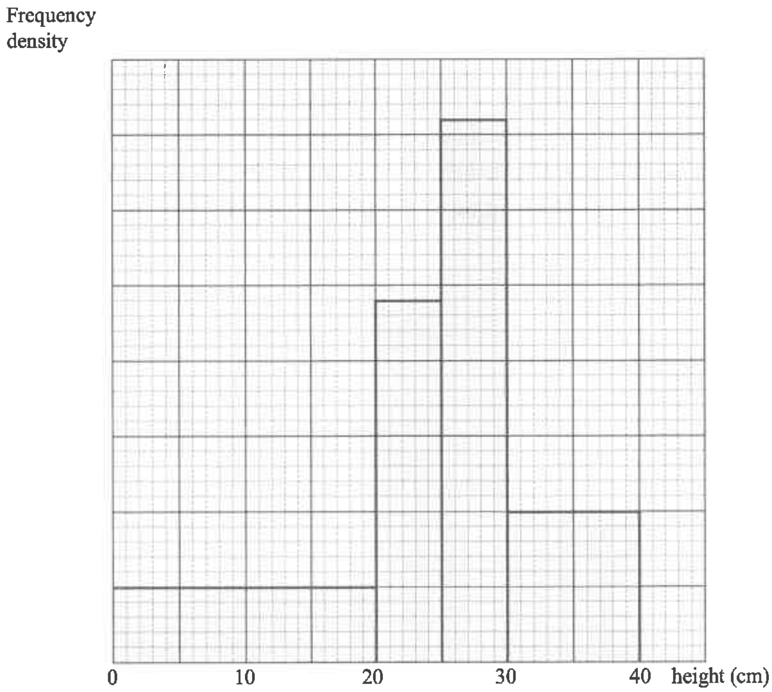
key 6 | 4 means 64 mph

(i) Describe shape of the distribution.
(ii) Find the median and quartiles.
(iii) Work out the mean and standard deviation.

72. The histogram shows the heights of 150 seedlings.

Complete the following table:

Length	Frequency
$0 \le x < 20$	
$20 \le x < 25$	
$25 \le x < 30$	
$30 \le x < 40$	



73. This frequency distribution shows the times taken by 30 students to complete a statistics test. Times are given to the nearest minute and you can assume that the times are evenly spread throughout each interval. Using linear interpolation: (i) calculate an estimate of the median time (to 3 s.f.)

Time (min)	Frequency
5 – 9	2
10 – 14	3
15 – 19	16
20 – 24	8
25 – 34	1

(ii) calculate an estimate for the interquartile range (to 3s.f.)

74. The discrete random variable X has the following probability distribution.

x	0	1	2	3
$P(X = x)$	¼	k	k + ¼	2k

(a) Find k
(b) Calculate F(2)

75. The 20 students on a Maths course recorded the time in minutes, x, to the nearest minute, spent travelling to college on a given day. The results are summarised here.
$\sum x = 513 \quad \sum x^2 = 14971.$
Find the mean and standard deviation for this data.

76. Five cards are numbered 0, 1, 1, 2 and 3. If S is the random variable 'the value of the card when one is picked at random'. Draw a table showing the probability distribution.

MECHANICS

Simple modelling:

Here is an example of a problem considered using a simple mathematical model.

Neil has a choice of doing his weekend shop at a supermarket 5 miles away, or at his local shop within walking distance. In a simple model he may consider price difference in the two shops and travelling expenses for the Supermarket.
He knows that the Supermarket prices are 10% cheaper than the local shop, but that petrol and running costs of the car could be estimated at 50p per mile.
Neil now has to apply his model.
If he spent £10 in the local shop, what would be the cost in the Supermarket?
Prices are 10% cheaper (£9) but add on travelling costs (5×50p) - total cost £11.50
So a trip to the Supermarket is not worth it.
If Neil planned to spend £30 at the local shop, what would be the cost in the Supermarket?
Cost £27 + 5×50p = £29.50 now the trip may seem worth it.

Mathematical models in mechanics:

(1) Take a real life problem.	*Shall Neil use the Supermarket?*
(2) Make some simplifying assumptions to produce a model you can deal with.	*Neil wants a simple model: he only considers 2 factors - price and travelling costs. He uses a general price comparison (assuming prices at the Supermarket are always 10% cheaper) and an estimate for travel costs per mile.*
(3) Present the problem in mathematical terms - that is produce a mathematical model.	
(4) Solve this problem (either theoretically or experimentally) and produce results.	*Neil tests his model with 2 possible shops - £10 and £30.*
(5) If you are satisfied with your solution you have finished, if not - go back to (2) and try improving your assumptions.	*Neil is happy that if he wants to spend £30 or more, a trip to the Supermarket is worthwhile.*

In Mechanics there are many models to help explain the effects of forces on objects. In the modelling you use in Mechanics you need to be aware of some frequently used vocabulary.

Basic Vocabulary for Modelling in Mechanics:

PARTICLE A particle is the name given to a body considered as a point in space, possessing mass but no size, sometimes called a "point mass".

Many objects in Mechanics problems are treated as particles, even when they clearly have dimensions, to make the modelling easier. "Particle mechanics" avoids complications such as air resistance and rolling.

LIGHT An object is light if its mass is so small, compared to the masses of other objects in the problem, that it can be considered as zero.
If an object with mass is suspended by a string, the string is normally considered to be light and its mass is ignored.

SMOOTH A smooth surface is one which offers so little resistance to another body sliding over it, that the friction is considered to be zero.
A smooth pulley is one which is considered to have no friction in its bearings. The tension in the string will be the same on either side of the pulley. A smooth peg is one where any contact with it is smooth and therefore no frictional forces act. So the tension in a string passing over it will be the same either side of the peg.

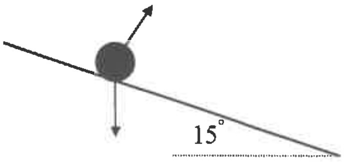
UNIFORM A body of uniform mass is one where equal amounts have equal masses.
Equal lengths of a uniform rod will have equal masses.
Equal areas of a uniform sheet of metal will have equal masses.

INEXTENSIBLE An inextensible (or inelastic) string is one whose length does not change even when acted upon by forces.

THIN A body is called thin when its thickness is very small compared to its width and length.
A piece of card or a sheet of metal would be considered thin. Each can be considered in a mechanics model as a two dimensional object.

RIGID A body is called rigid if it keeps its shape when acted on by forces.

Setting up simple models in mechanics

A slate sliding down a roof, inclined at 15° to the horizontal. Assumptions: The slate can be modelled as a particle. The roof can be modelled as a smooth plane. The slate moves in a vertical plane.  Once the forces have been added to the diagram, F = ma and then the equations of motion with constant acceleration can be used.	*You can draw a 2 dimensional diagram if the slate is moving in a vertical plane.*
A tennis ball in flight. Assumptions: The ball can be modelled as a particle. No air resistance acts on the ball. The ball moves in a vertical plane.  The only force acting on the ball is its weight. F = ma can then be used and the equations of motion with constant acceleration.	*With these assumptions you ignore any spin on the ball or the effect of turbulence.*
82. A rope at 30° above the horizontal is pulling a boat along a horizontal chute. The rope passes over a pulley and is being pulled vertically downward with a force of 500 Newtons. Assumptions:	*Try this one yourself.* *List the assumptions and then draw a diagram showing what forces are acting.*

MOTION IN ONE DIMENSION:

Displacement is the change in position. It is a vector and therefore has magnitude and direction. In straight line motion an object can be forwards or backwards from its initial position and therefore a +ve or –ve value will show the direction.

If an object is travelling with constant speed:

$$speed = \frac{distance}{time}$$

If speed is not constant:

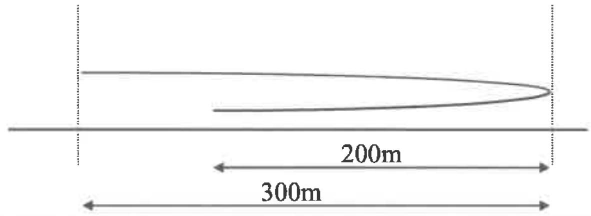
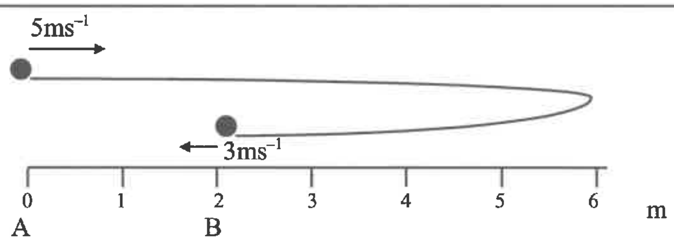
$$average\,speed = \frac{total\ distance}{total\ time}$$

Speed is a scalar quantity.

Velocity is a vector quantity and therefore has magnitude (speed) and direction. In straight line motion the velocity can be in one of two directions i.e. +ve or –ve.

$$velocity = \frac{displacement}{time} \qquad average\ velocity = \frac{total\ displacement}{total\ time}$$

Velocity is the rate at which the position of an object changes.
e.g. a body changes from position 3 m to position 11 m in 2 seconds. Its change in position is 8 m in 2 seconds i.e. average velocity + 4 ms $^{-1}$ (4 metres per second in the +ve direction).

83. A woman walks 300 m due East in 100 seconds, then 200 m due West in 80 seconds. Calculate her average speed and her average velocity. Diagram:  Complete: Total distance travelled = Total displacement = Average speed = Average velocity =	*Take East as the positive direction.* *Remember distance just means ground covered in this case.* *Displacement means position relative to starting point.* *Just use the formulae above.* *Give answers to 3 sig.fig.* *Don't forget to state the units used and the direction.*
84.  A body at A moves as shown to B. Distance travelled from A to B = Displacement of B from A = Speed of body at A = Velocity of body at A = Speed of body at B = Velocity of body at B =	***Assume this is straight line motion.*** *Take 'right as the positive direction.* *Indicate the direction by + or – sign for displacement and velocity.*
85. Convert 40 km h $^{-1}$ to m s $^{-1}$	*First change the km to m.* *Then change hours to seconds.*
86. Convert 4 m s $^{-1}$ to km h $^{-1}$	*First change the seconds to hours.* *Then change m to km.*

87. P, Q and R are three points, in that order, on a straight road with PQ = 6 km and QR = 8 km. A man travels from P to Q at 12 km h^{-1} and from Q to R at 20 km h^{-1}. Calculate:
(i) the time taken to travel from P to Q
(ii) the time taken to travel from Q to R
(iii) the average speed of the man in travelling from P to R.

You can use:

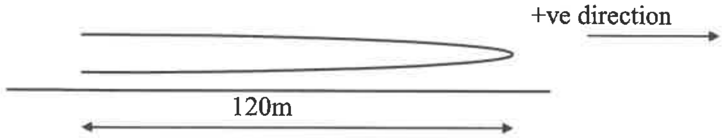
$$speed = \frac{distance}{time}$$

as we are assuming constant speed.

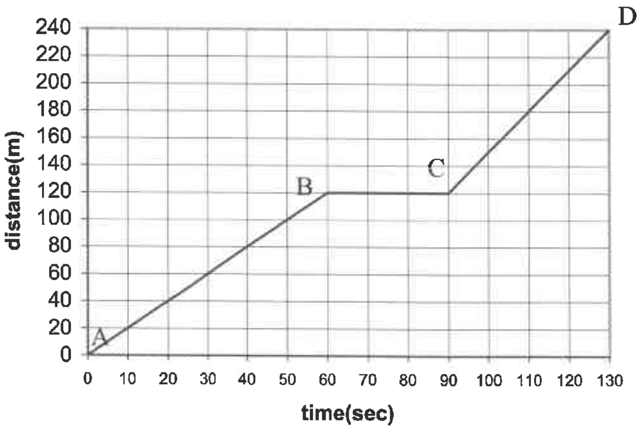
(iii) Answer to 3 sig. fig.

Distance/position time graphs:

Gill walks at constant speed to her friend's house 120 m away in 60 seconds. After talking to her friend for 30 seconds she returns home at constant speed in 40 seconds.



Distance time graph.



A to B 120 m in 60 sec
i.e. speed 2 ms^{-1}

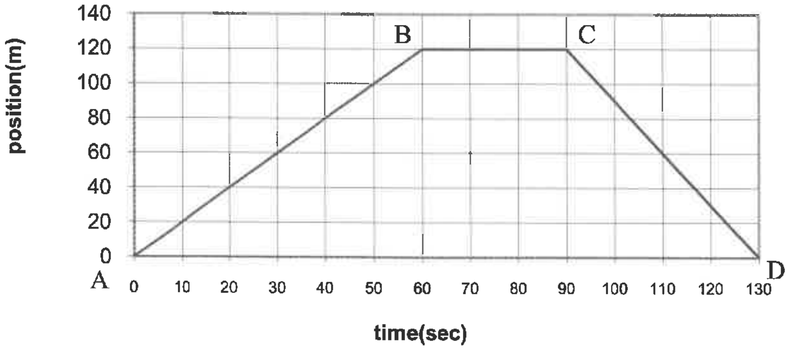
Equal to the gradient from A to B

B to C waiting (zero speed zero gradient)

C to D 120 m in 40 sec
i.e. speed 3 ms^{-1}

Equal to the gradient from C to D

Position time graph. If the position is related to a fixed point or origin, this can also be called a displacement time graph.



A to B 120 m in 60 sec
i.e. velocity 2 ms^{-1}

Equal to the gradient from A to B

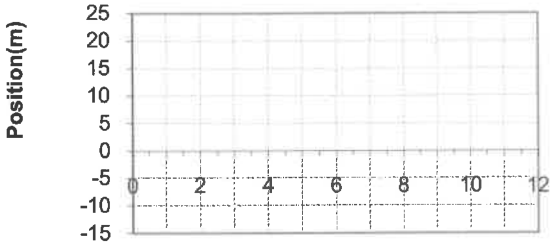
B to C waiting (zero velocity zero gradient).

C to D 120 m in 40 sec (going back to home).
i.e. velocity -3 ms^{-1} (negative indicates velocity in the opposite direction).

Equal to the gradient from C to D

The gradient of a position (displacement) time graph is equal to the velocity.

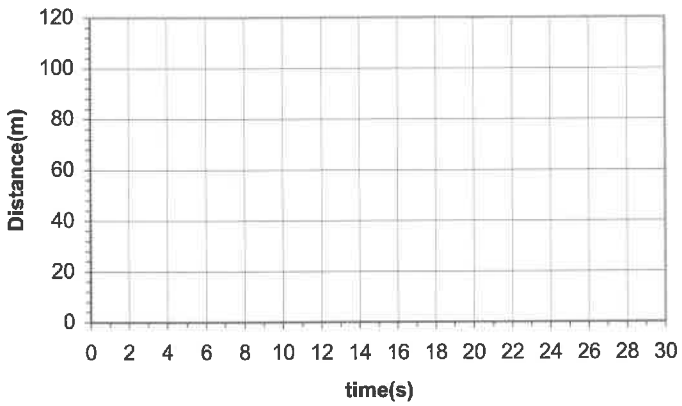
88. A footballer is doing some fitness training. Starting from A he runs at constant velocity to B in 4 seconds. He then immediately turns around and runs at constant velocity to C in 8 seconds. Sketch the position time graph.

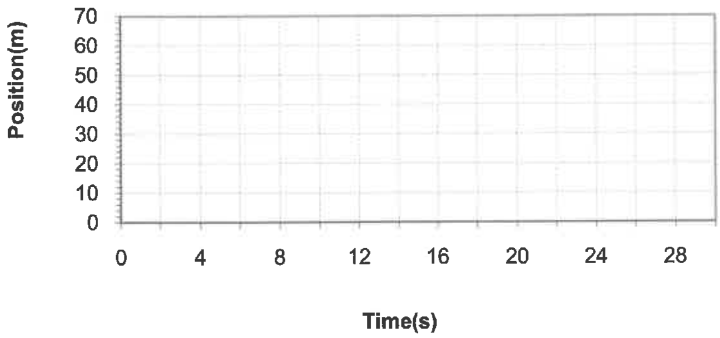


Work out the velocities for the two parts of the motion.

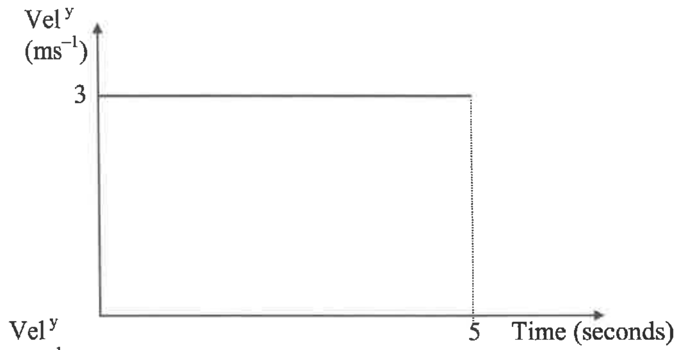
Assumption: Constant velocities for each part of the motion.

89. A goods truck is shunted 50 m forward in 10 seconds, then 28 m back in 7 seconds and then 36 m forward in 12 seconds.
(i) Sketch the distance-time graph.
(ii) Calculate the speeds for each section.
(iii) Find the average speed for the whole journey.
(iv) Sketch the position-time graph.
(v) Calculate the velocity for each section.
(vi) Find the average velocity for the whole journey.

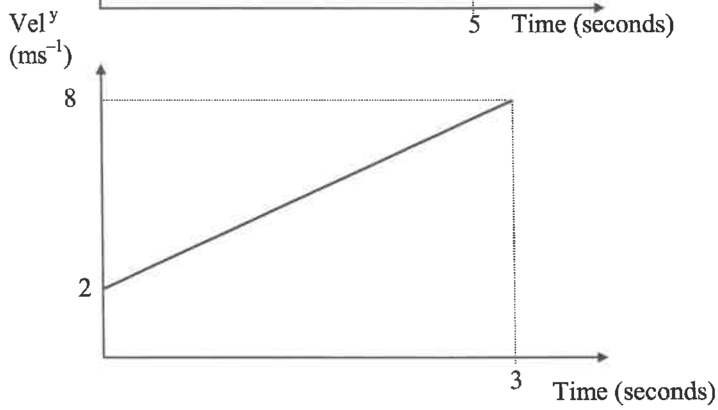




Velocity time graphs:



This graph shows constant velocity of 3 ms^{-1} for 5 seconds. The body travels $3 \times 5 = 15$ m. This is also the area under the graph.



Acceleration is the rate at which the velocity changes.
Here the velocity changes from 2 ms^{-1} to 8 ms^{-1} in three seconds.
The velocity changes by 6 ms^{-1} in 3 seconds which is $\frac{6}{3} = 2$ ms^{-1}per sec
i.e. the acceleration is 2 ms^{-2}.
This is the gradient of the line.

The area between a velocity time graph and the time axis is equal to the distance. For ***distance*** add the areas. For ***displacement*** add the areas above the time axis and subtract the areas below the time axis.

The gradient of a velocity time graph is equal to the acceleration.

90. A particle starting from rest accelerates uniformly in a straight line for 4 seconds, by which time it is travelling at 14 ms^{-1}. It then travels at this velocity for 3 seconds before uniformly returning to rest in 5 seconds.

(a) Draw a sketch of the velocity-time graph for this particle.

Only a sketch. No need for a scale. Use a ruler and mark the axes at the important points.

For part (b) we could use the equations of motion – but using the graph is easier.

(b) Find the accelerations of the particle for the three parts of the journey and the total distance travelled.

Complete:

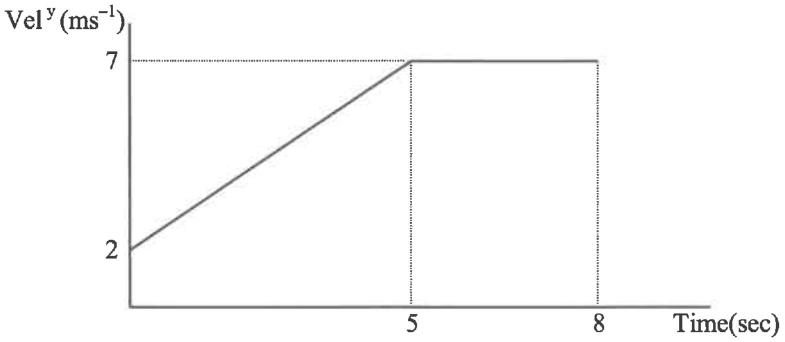
1st part Constant acceleration of ms^{-2} for 4 seconds. *Gradient of graph.*

2nd part Constant acceleration of ms^{-2} for 4 seconds. *Gradient of graph. (Note: negative i.e. negative acceleration which is deceleration.)*

3rd part Constant acceleration of ms^{-2} for 5 seconds. *Area under graph. (Area of a trapezium or two triangles and a rectangle.)*

Total distance travelled =

91. This velocity time graph shows a body which accelerates uniformly for 5 seconds, after which time it maintains its final velocity for 3 seconds.
Find (i) the initial velocity, u
(ii) the final velocity, v
(iii) the acceleration, a, during the first 5 seconds
(iv) the distance travelled by the body during the 8 seconds.



$u =$

$v =$

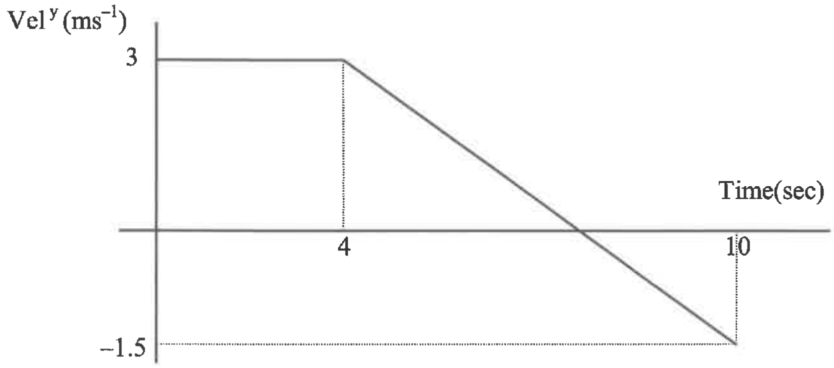
$a =$

distance =

Gradient of the graph.

Area between graph and the t axis.

92. This velocity time graph shows a body moving at constant velocity for 4 seconds and then decelerating uniformly for 6 seconds. Find
(i) the initial velocity, u
(ii) the final velocity, v
(iii) the acceleration, a, during the last 6 seconds
(iv) the displacement from its starting point
(v) the total distance travelled by the body.



(i) $u =$

(ii) $v =$

(iii) $a =$

(iv) Displacement =

(v) Distance =

Acceleration is negative.

To find the distance and displacement you will need to work out where the line cuts the time axis.
Use areas.
For displacement: above the time axis is +ve and below is –ve. Then just add.

For distance: make both the areas +ve and add.

93. A particle travelling in a straight line has an initial velocity of $10\,\text{ms}^{-1}$ and an acceleration of $-2\,\text{ms}^{-2}$
(a) Draw a velocity-time graph for the first 7 seconds of motion.
(b) Find the displacement from the start after 7 seconds.
(c) Find the distance travelled in 7 seconds.

Only a sketch. No need for a scale. Use a ruler and mark the axes at the important points.

Displacement =

Use areas. Above the time axis is +ve and below is –ve. Then just add.

Distance travelled =

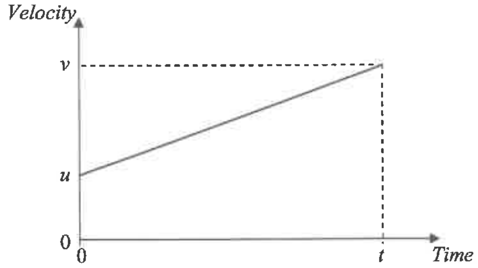
Make both the areas +ve and add.

94. A cyclist rides along a straight road from P to Q. He starts from rest at P and accelerates uniformly to reach a speed of $15\,\text{ms}^{-1}$ in 10 secs.
He maintains this speed for 20 seconds, then slows down uniformly to stop at Q. The whole journey takes 36 secs.
(i) Sketch a velocity-time graph for the journey.
(ii) Calculate the acceleration for the first part of the journey.
(iii) Calculate the deceleration for the last part of the journey.
(iv) Find the total distance travelled by the cyclist.

95. Two cars Z and X start from rest, travel equal distances in the same time and finish at rest. Car Z accelerates at $1.25\ ms^{-2}$ for 16 seconds, travels at constant speed for 20 seconds and is then brought to rest with a uniform deceleration of $2\ ms^{-2}$. Car X accelerates uniformly at $x\ ms^{-2}$ and is then brought to rest with a uniform deceleration of $3x\ ms^{-2}$. Sketch the velocity-time graphs of the two cars on the same diagram and calculate: (i) The greatest speed reached by Z. (ii) The total distance travelled by Z. (iii) The time for which X accelerates. (iv) The greatest speed reached by X.

Motion in a straight line with constant acceleration:

Derivation from velocity time graph:



Consider a object moving for t seconds at constant acceleration a units. If the initial velocity is u units and the final velocity is v units, a velocity-time graph can be drawn as shown.

Gradient of graph = acceleration, $a = \dfrac{v-u}{t}$ $\therefore$ $v = u + at$

Area under the graph = Displacement, $s = \frac{1}{2}t(u+v)$ (from the area of a trapezium)

96. Using $v = u + at$....(i) and $s = \frac{1}{2}t(u+v)$....(ii) derive the following:(a) $s = ut + \frac{1}{2}at^2$ (b) $s = vt - \frac{1}{2}at^2$ (c) $v^2 = u^2 + 2as$	*(a) Substitute (i) into (ii)* *(b) Rearrange (i) to give u = and then substitute into (ii)* *(c) Rearrange (i) to give t = and then substitute into (ii)*

The five possible **equations of motion for constant acceleration** are as follows:

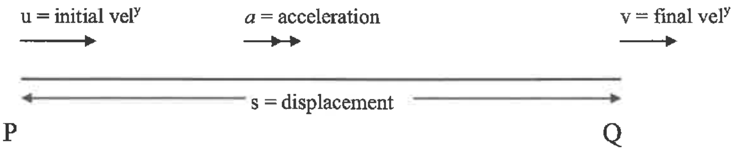
$$v = u + at$$
$$s = ut + \tfrac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

Where:
s = displacement
u = initial velocity
v = final velocity
a = uniform acceleration
t = length of time interval

$$s = \tfrac{1}{2}t(u + v)$$
$$s = vt - \tfrac{1}{2}at^2$$

Only use these two equations if you are used to using them.

97. A body is moving in a straight line from P to Q.



Careful in all questions. $speed = \dfrac{distance}{time}$ *is for zero acceleration (constant velocity). Do not use it for these questions.*

If:

(a) u = 0, $a = 7$, t = 8 , find s

(b) u = 5, $a = 3$, t = 5 , find v

(c) Initially at rest, $a = 2$, t = 5, find s

(d) u = –5, v = 7, t = 10, find s

(e) u = 7, v = 9, s = 2, find a

(f) v = 32, $a = 6$, t = 3, find the initial velocity

(g) u = 17, v = – 13, s = 10, find a

(h) s = 32, $a = 2$, t = 4, find u

(i) s = 144, t = 6, $a = -2$, find v

(j) u = 5, t = 10, $a = -3$, find s

(k) u = 6, v = –2, t = 6, find a

98. A dog accelerates from rest to 15 ms^{-1} in a distance of 30 m. Find its acceleration (assume constant).

Write down $u =$ $v =$ $s =$ *and* $a = ?$
Write down the equation you will use.
Then use it to find a.

99. A car decelerates uniformly from 20 ms^{-1} until it stops after 6 seconds. What is its deceleration?	*Write down $u =$ $v =$ $s =$ and $a = ?$ Write down the equation you will use. Then use it to find a. This will be negative.*
100. A car accelerates uniformly from rest to 96 km h^{-1} (about 60 mph) in 12 seconds. Find its acceleration and the distance travelled during acceleration	*Change to m s^{-1}.* *Write down $u =$ $v =$ $t =$ and $a = ?$ Write down the equation you will use. Then use it to find a.* *Then find s.*
101. An electric hare travelling on a straight track at 5 ms^{-1} passes a stationary greyhound. One second later the greyhound sets off after the hare, accelerating at 4 ms^{-2}. How long is it before the greyhound catches up with the hare, from the time when the hare passes the greyhound, and how far has the greyhound travelled?	*If the hare has been moving for t seconds since passing the greyhound then the greyhound has been moving one second less i.e. for (t–1) seconds.* *Use '$s = ut + \frac{1}{2}at^2$' for both (with (t–1) for t for the greyhound) and equate.*
102. A boy on a sledge pushes himself off, at the top of a slope with a speed of 1.5 ms^{-1}. He moves with constant acceleration and after 10 seconds is moving at 4 ms^{-1}. (i) What is the acceleration of the boy and sledge? (ii) The length of the slope is 200 m. What is the speed of the boy and sledge at the bottom of the slope?	*Sketch. Show the initial speed, the distance, final speed and the acceleration.* *Use the constant acceleration equations.*

Vertical motion:

This is motion in a straight line (up and down) with constant acceleration due to gravity. Use $g = 9.8\ \text{ms}^{-2}$ unless told otherwise. **Use the equations of motion**, as before, but this time you know the acceleration.

Note: (i) If an object is thrown up, at the top of its flight its velocity is zero.
The velocity of a particle *as it hits the ground* is NOT zero (think of it as the velocity just before it hits the ground).
Choose a positive direction (Usually up so, $a = -9.8$)

103. You throw a ball vertically upwards with an initial velocity of $15\ \text{ms}^{-1}$.
   (i) Find the time taken for the ball to reach the top of its trajectory.
   (ii) Find the time taken for the ball to return to your hand.
   (iii) If you do not attempt to catch the ball and it hits the ground 1.2 m below your hand, how long has it been in the air?
   (iv) How fast does it hit the ground?

Model the ball as a particle.

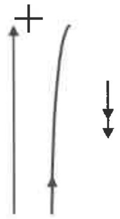
Units will be ms^{-1}, m, s and ms^{-2}

Answers to 3 sig. fig.

Complete:

(i) Up positive: $u = 15 \quad v = 0 \quad a = -9.8 \quad t = ?$
Use $v = u + at$

$0 = 15 + (\qquad)t \Rightarrow t = \qquad$ secs.



The initial velocity is given as $15\ ms^{-1}$ and the ball was thrown up, so up must be the positive direction.
List the values you know and the one you want to find. This should help you decide which equation of motion you need to use.

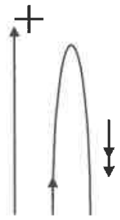
(ii) Up positive: $u = 15 \quad s = 0 \quad a = -9.8 \quad t = ?$
Use $s = ut + \frac{1}{2}at^2$

$0 =$

$0 = t(15 - 4.9t)$

$\Rightarrow t = 0$ (when ball is thrown) or $t = \qquad = \qquad$ secs

So ball returns to hand level after $\qquad$ secs.



Displacement at the hand is 0 : this will happen twice – first when the ball is thrown and again when the ball falls back to its initial position
Factorise. t is a common factor.

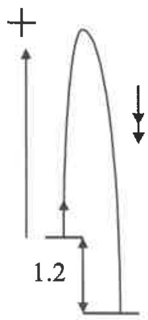
(iii) Up positive: $u = 15 \quad s = -1.2 \quad a = -9.8 \quad t = ?$
Use $s = ut + \frac{1}{2}at^2$

Position is 1.2 m below the initial position – therefore if up is +ve then $s = -1.2$

$-1.2 = 15t - 4.9t^2 \Rightarrow 4.9t^2 - 15t - 1.2 = 0$

$t =$

Use the quadratic formula.
Use the positive root.



(iv) Up positive: $u = 15 \quad s = -1.2 \quad a = -9.8$
Use $v^2 = u^2 + 2as$

$v^2 = 225 + 2(\qquad)(\qquad) =$

$v =$

Square root.

104. A ball is dropped from the top of a cliff. 1 second later a stone is thrown vertically down from the top of the cliff at 15 ms $^{-1}$. The ball and the stone hit the sea at the same time. Find the height of the cliff. Complete: Ball: $u = 0$, $a = 9.8$, $t = \text{T}$ $s = ut + \frac{1}{2}at^2$ $s =$ Stone: $u = 15$, $a = 9.8$, $t = \text{T} - 1$ $s = ut + \frac{1}{2}at^2$ $s = 15(\text{T}-1) + 4.9(\text{T}-1)^2$	*Take down as positive.* *If the ball is in the air for T seconds, the stone is in the air for 1 second less i.e. T – 1 seconds.* *Careful* $(T-1)^2 = (T-1)(T-1)$ *Distance the same, so equate to find the time when they are together. Use this time to find the height (the displacement) at this time.* *Answer to 3 sig. fig.*
105. A book falls from a shelf 2 m above the floor. At what speed does it hit the floor?	*Take down as positive.* $u = 0$ $s = 2$ $a = 9.8$ *Use* $v^2 = u^2 + 2as$
106. A stone is dropped from a cliff 70 m above the sea. How long is it in the air before it hits the sea?	*Take down as positive.* $u = 0$ $s = 70$ $a = 9.8$ $s = ut + \frac{1}{2}at^2$ *Answer to 3 sig. fig.*
107. A brick is dislodged from a hod on some scaffolding 25 m above the ground. Its initial velocity is 0.5 ms $^{-1}$ downwards. Find the velocity with which it hits the ground and the time it takes to reach the ground.	*Take down as positive.* $u = 0.5$ $s = 25$ $a = 9.8$ *Use* $v^2 = u^2 + 2as$ *for final velocity* *Then* $v = u + at$ *or* $s = ut + \frac{1}{2}at^2$ *for time to fall to ground.* *Answer to 3 sig. fig.*
108. A stone is thrown upwards with a speed of 20 ms $^{-1}$. Find the length of time the stone is above a height 10 m, if (i) the stone is thrown from ground level (ii) the stone is thrown from 2 m above ground level.	*Use* $s = ut + \frac{1}{2}at^2$ *to find the 2 times when the stone is 10 m above the ground.* *Take up as positive.* *For (ii) If the stone is thrown from zero displacement, you need the times when the displacement from the initial position is 8 m. Find the 2 times when the stone has a displacement of 8 m.* *Answer to 3 sig. fig.*

Position, velocity or acceleration in terms of time:

If position, velocity or acceleration of a particle is given to you in terms of time,
e.g. displacement, $x = 3t^2 + 2t - 5$ or velocity, $v = 2t^2$
then use:

velocity,	$v = \frac{dr}{dt}$	**r** is the position vector of the particle. Differentiate **r** with respect to t. e.g. If $\mathbf{r} = t^2$ then $v = \frac{dr}{dt} = 2t$
acceleration,	$a = \frac{dv}{dt}$	Differentiate **v** with respect to t. e.g. If $v = 3t^2$ then $a = \frac{dv}{dt} = 6t$

and

velocity,	$v = \int a\, dt$	Integrate a with respect to t. e.g. If $a = 3t$, then, $v = \int 3t\, dt = \frac{3t^2}{2} + c$
position,	$r = \int v\, dt$	Integrate **v** with respect to t. e.g. If $v = t^2$, then, $\mathbf{r} = \int t^2\, dt = \frac{t^3}{3} + c$

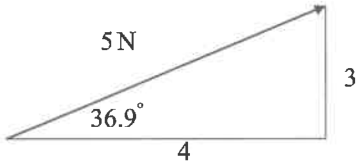
DON'T FORGET THE CONSTANTS OF INTEGRATION.

109. COMPLETE: If displacement, $s = 5t^2 - 4t + 3$	
velocity, $v = \frac{ds}{dt} =$	*Differentiate.*
acceleration, $a = \frac{dv}{dt} =$	*Differentiate.*
110. COMPLETE: If acceleration, $a = 5t - 3$ and $s = 0$ and $v = 5$ when $t = 0$	
velocity, $v = \int a\, dt = \int \quad dt$	*Integrate.*
$=$	*Don't forget the constant of integration.*
$v = 5$ when $t = 0$ $\therefore$ $5 =$	*Find the constant -use the information given.*
$\therefore$ $v =$	*Write the formula for v with the constant.*
displacement, $s = \int v\, dt = \int \quad dt$	*Integration.*
$=$	*Don't forget the constant of integration.*
$s = 0$ when $t = 0$ $\therefore$ $0 =$	*Find the constant -use the information given.*
$\therefore$ $s =$	*Write the formula for s with the constant.*

Remember: If asked for maximum or minimum then differentiate and equate to 0.

Resolving forces:

A force is a vector: it has magnitude (size) and direction. When working with a force we can write it in different ways.



Magnitude / direction form

↓

5 Newtons at 36.9° to horizontal

Component form

↓

$(4\mathbf{i} + 3\mathbf{j})$ Newtons or $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ Newtons

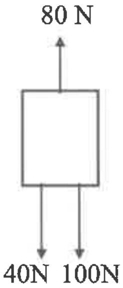
horizontal component $5\cos 36.9^\circ = 4$
vertical component $5\sin 36.9^\circ = 3$

The resultant force:

For forces acting in one dimension (along a straight line) just add and/or subtract the forces to give the resultant force. Make one direction positive. State the direction of the resultant.

Resultant force = 5 – 2 = 3 N in the 5 N direction.

Resultant force = 6 + 8 – 12 = 2 N in the 6 N direction.



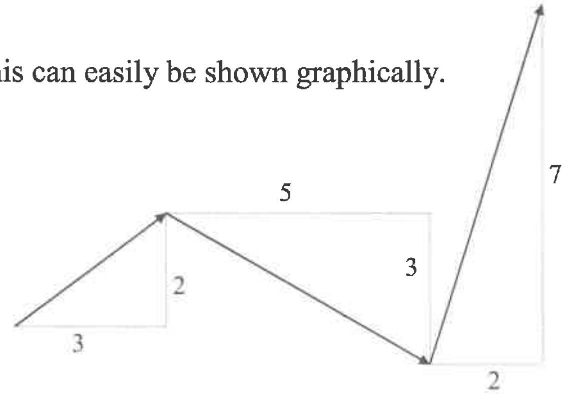
Resultant force = 100 + 40 – 80 = 60 N downwards.

For forces given in component form, simply add and/or subtract.

The resultant of the forces $(3\mathbf{i} + 2\mathbf{j})$N, $(5\mathbf{i} - 3\mathbf{j})$N and $(2\mathbf{i} + 7\mathbf{j})$N is:

$$3\mathbf{i} + 2\mathbf{j} + 5\mathbf{i} - 3\mathbf{j} + 2\mathbf{i} + 7\mathbf{j} = (10\mathbf{i} + 6\mathbf{j})\text{ N} \qquad \text{or as column vectors} \quad \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ -3 \end{bmatrix} + \begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}\text{ N}$$

This can easily be shown graphically.



In **i** direction 3 + 5 + 2 = 10

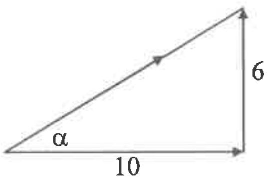
In **j** direction 2 – 3 + 7= 6

Magnitude = $\sqrt{100+36} = \sqrt{136} = 11.7$ to 3 sig.fig.

$\tan\alpha = \frac{6}{10} = 0.6$

$\alpha = 31.0^\circ$

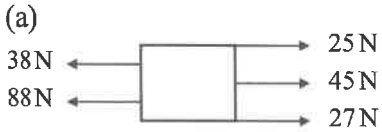
The direction is 31.0° anticlockwise from the positive x direction.



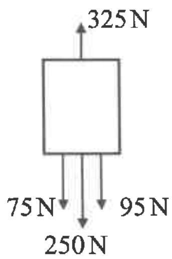
122. Find the resultant of the forces $(-5\mathbf{i} + 2\mathbf{j})$N, $(8\mathbf{i} + 3\mathbf{j})$N and $(2\mathbf{i} - 7\mathbf{j})$N.	*Just add.*

123. Work out the resultant force in each of the following cases.

(a)

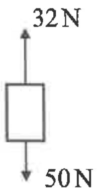


(b)



For forces acting along a straight line. Make one direction positive and add.

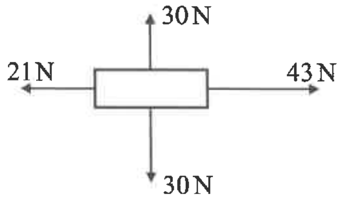
124. Find the resultant of these forces.



A simple question.

Don't forget the direction.

125. Find the resultant of these forces.



Resolving in the x direction

Resolving in the y direction

Answer:

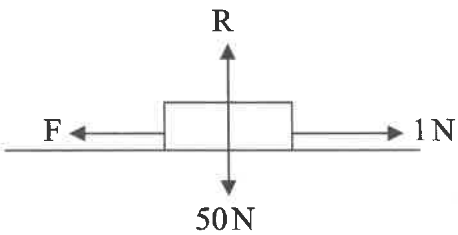
A simple question. The forces are already in the x and y directions. Just sum in each of these directions.

Be clear in your answer.

Friction:

- Friction opposes motion

Consider an object being pulled.
In equilibrium:
Pulling force 1 N

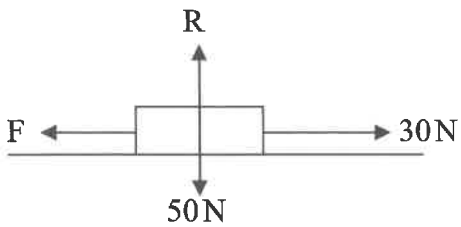


In equilibrium
∴ R = 50 N
F = 1 N

and so on (increasing the pulling force) until object is on the point of moving - <u>limiting equilibrium</u>

Moving object:
Pulling force 30 N

Friction, F = 25 N



Vertically:
no resultant force.
∴ R = 50 N
Horizontally:
F = 25 N
and so
Resultant force = 30 – 25
= 5 N
∴ the object has a resultant force acting on it and so it is accelerating.

Newton's First Law of Motion:

A body will continue in a state of rest or move at a constant velocity, unless acted upon by an external force. In other words a change in the velocity of a body is caused by a force.

Equilibrium:

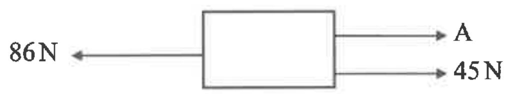
A body at rest: - if forces act on the body and it does not move the forces must be "in equilibrium" - they balance out.
The resultant force IN ANY DIRECTION is zero.

A moving body: - If the body is moving at a **constant velocity** there is no acceleration and so there is no resultant force acting on it. i.e. the forces must be "in equilibrium" - they balance out.
The resultant force IN ANY DIRECTION is zero.
If a resultant force acts on a body its velocity will change: the body may speed up, slow down **or change direction**.

> If a set of forces are in **equilibrium** the RESULTANT FORCE (sum of the forces) must be zero. And so the sum of their components, in any direction, must also be zero.

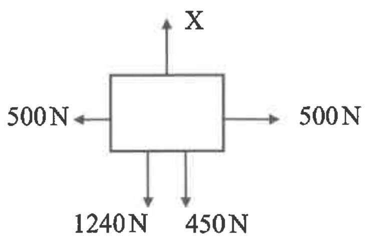
Forces acting in one dimension.

<table>
<tr><td>126. A body is at rest under the action of the forces shown in the diagram. Fill in the value of A.



</td><td>The sum of the forces is zero (make one direction +ve).
OR
The forces acting to the right equals the forces acting to the left.</td></tr>
<tr><td>127. A body moves horizontally at a constant 4 ms^{-1} under the action of the forces shown.
Find X

X =



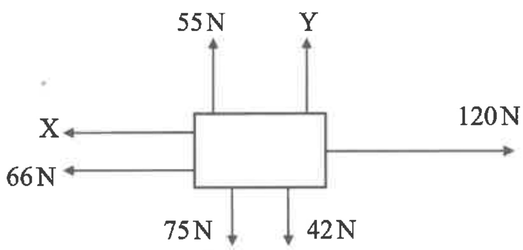
</td><td>The sum of the forces is zero (make one direction +ve).
OR
The forces acting up equals the forces acting down.</td></tr>
</table>

Forces acting in 2 dimensions.

<table>
<tr><td>128. A body moves horizontally at a constant 4 ms^{-1} under the action of the forces shown. Find X and Y.

X =

Y =



</td><td>No acceleration in any direction. Therefore the forces are in equilibrium.</td></tr>
</table>

For forces given in component form.

<table>
<tr><td>Example: The forces $(6\mathbf{i} - 4\mathbf{j})$ N, $(-2\mathbf{i} + 3\mathbf{j})$ N and $(x\mathbf{i} + y\mathbf{j})$ N are in equilibrium. Find x and y.

Parallel to x axis (in direction of $\mathbf{i}$): $6 - 2 + x = 0$ $x = -4$
Parallel to y axis (in direction of $\mathbf{j}$): $-4 + 3 + y = 0$ $y = +1$</td><td>The forces are already split into components here. A diagram is not necessary.</td></tr>
</table>

129. A body of mass 20 kg, is being pulled along the floor with constant velocity by a horizontal force of magnitude 140 N. Find the normal reaction acting on the box and the frictional force between the body and the floor.	*Draw a diagram.* *Constant velocity – therefore the forces are in equilibrium.* *Remember, weight = mg*
130. A person of mass 45 kg is in a lift. The lift is going vertically upward with a constant velocity. What is the reaction force of the lift on the person?	*Draw a diagram. Put in the forces acting on the person.* ***Constant velocity** - therefore the forces are in equilibrium.*
131. Forces $2\mathbf{i}+4\mathbf{j}$, $5\mathbf{i}-q\mathbf{j}$, $-2\mathbf{i}+5\mathbf{j}$ and $p\mathbf{i}-6\mathbf{j}$ Newtons are in equilibrium. Find the value of p and q.	*In equilibrium. Therefore the sum of the forces equals zero.* *Total of **i** components must be zero.* *Total of **j** components must be zero.*
132. Each of these sets of forces is in equilibrium. Find the values of x and y in each case. (i) $(5\mathbf{i}+3\mathbf{j})$N, $(2\mathbf{i}-7\mathbf{j})$N, $(x\mathbf{i}+y\mathbf{j})$N (ii) $(-3\mathbf{i}-8\mathbf{j})$N, $(x\mathbf{i}+3\mathbf{j})$N, $(-2\mathbf{i}+y\mathbf{j})$N (iii) $(\mathbf{i}-\mathbf{j})$N, $(3\mathbf{i}-y\mathbf{j})$N, $(-5\mathbf{i}-y\mathbf{j})$N, $(x\mathbf{i}+5\mathbf{j})$N	*Total of **i** components must be zero.* *Total of **j** components must be zero.*

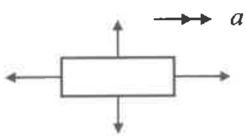
Newton's Second Law of Motion:

Simply **Resultant force, F = m *a***

i.e. Resultant force in any direction equals mass times acceleration in that direction.

In problems you may have to find the acceleration from a given force system, so that you can then use the constant acceleration formulae to work out a displacement, velocity or time. In others you may have to use the constant acceleration formulae to work out an acceleration and hence find a force by using F = m *a*.

Forces acting in one direction.

133. Find the acceleration experienced by a particle of mass 6 kg, when acted on by a resultant force of 25 Newtons.	*Just use F = ma.*
134. What resultant force will produce an acceleration of 0.5 ms^{-2} in a car of mass 850 kg?	*Just use F = ma*
135. Assuming acceleration to be constant, find the force that increases the velocity of a car of mass 750 kg from 2 ms^{-1} to 8 ms^{-1} in 3 seconds.	*First find the acceleration using one of the constant acceleration formulae.* *Then use F = ma.*
136. A vehicle of mass 550 kg moving at 12 ms^{-1} is brought to rest in 5 m. Find the braking force.	*First find the acceleration using one of the constant acceleration formulae.* *Then use F = ma.*
137. A trailer of mass 400 kg is being pulled along a straight road with a force of 550 N. If the resistance to motion is 375 N, find the acceleration of the trailer. 	*A sketch may help.* *No resultant force vertically and friction is in the resistance to motion. So no need to consider the vertical forces.* *Remember resultant force = ma.*
138. The tension in a rope being used to raise an object is 61.5 N. If the acceleration of the object is 2.5 ms^{-2}, find the mass of the object.	*Sketch.* *Resultant force = ma.* *Terms with m on one side of the equation, so you can take m out as a factor.*
139. A bullet of mass 25 grams is fired horizontally into a fixed wooden block, which offers a constant resistance of 1000 N to the motion of the bullet. Find the deceleration of the bullet.	*Newtons are kg m s^{-2} so the mass must be in kg.* *Then use F = ma.*

Forces acting in two perpendicular directions.

140. A particle of mass 3 kg is acted on by a force of (3**i** + 7**j**) N. Find the magnitude of the resulting acceleration to 3 significant figures.

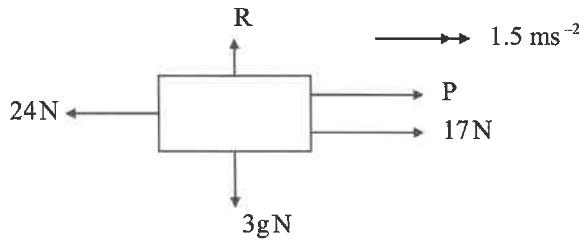
*Use **F** = ma. **F** and **a** are vectors, so just put the vector for **F** in this equation and the acceleration will be a vector in this form.*

141. Find the magnitude and direction of the resultant force required to give a particle, of mass 5 kg, an acceleration of (5**i** – 2**j**) ms^{-2}.

*Use **F** = **ma** to find **F** as a vector. First sketch the force vector. Then find the magnitude and direction.*

Answer to 3 sig. fig.

142. Forces act on a body, as shown, giving rise to the acceleration, also shown. Find the vertical force R and the horizontal force P.

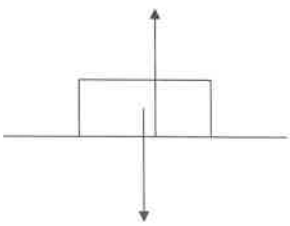


143. A particle of mass 8 kg is being pulled along a rough horizontal surface by a string parallel to the surface. If the tension in the string is 74 Newtons and the resistance to motion is 6g Newtons (where g is acceleration due to gravity), find the acceleration of the particle.

Sketch.
Put in the forces: mg , R (the normal reaction), T (tension) and Fr (resistance to motion).

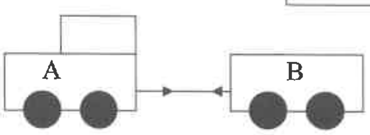
Vertically: forces in equilibrium.
Horizontally:
Resultant Force = ma

Newton's Third Law of Motion:
If a body, A, exerts a force on a body, B, then B exerts an equal and opposite force on A.



Body at rest. Body presses down on the ground and the ground pushes up on the body (reaction).

Body pulling down on the ceiling via string (tension) and the ceiling holding up the body via string (tension).



Body A pulling on B via string (tension) and Body B pulling on A via string (tension).

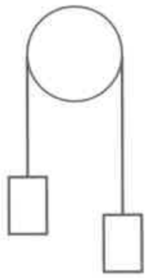
Connected bodies:

Remember: For a smooth pulley, or a smooth peg, the tension in the string on either side of the pulley or the peg is the same.

144. Two sacks of grain, of masses 35 kg and 25 kg, are connected by a light inextensible string passing over a smooth pulley. If the sacks are released from rest, find the acceleration of the sacks and the tension in the string.

For the 35 kg sack:

For the 25 kg sack:



Remember weight is mg

Put in the forces acting on each sack. mg and the tension.

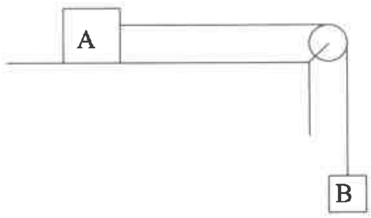
Show the acceleration.

Use resultant force = ma for each sack.

Solve the simultaneous equations to find the acceleration and the tension.

Answers to 3 sig. fig.

145. Two objects A and B of mass 3 kg and 2 kg respectively, are connected by a string. Object A is held on a smooth horizontal surface and the string passes over a pulley so that object B hangs freely. Object A is released. Find the acceleration of B and the tension in the string. List any other assumptions being made.



Smooth so no friction.

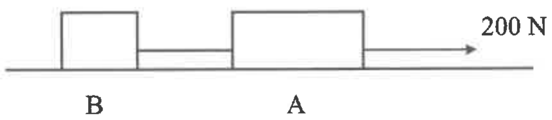
Put in the forces acting on each object. mg, tension and normal reaction for A, mg and the tension for B.

Show the acceleration of A and B on the diagram. The same for both B down and A.

Use resultant force = ma for each object.

The weight (mg) and normal reaction of A are not used in these calculations.

146. Two blocks attached together by string, are being pulled along a horizontal straight line by a force of 200 N, as shown below. If block A has mass 7 kg and the resistance to its motion is 60 N and block B has mass 5 kg and the resistance to its motion is 40 N, find the acceleration of the two blocks and the tension in the string.



Put in the forces acting on each block.

Tension in string pulling back on A and pulling forward on B.

Show the acceleration of A and B on the diagram. (The same for both).

Use resultant force = ma for the whole system.

Then use resultant force = ma for either A or B to find the tension in the string.
Answers to 3 sig. fig.

Mixed questions 2

147. Two motorcycles A and B, travel equal distances of 10 km in the same time of 20 minutes, finishing at rest. A starts from rest, accelerates uniformly to a speed of 40 km h^{-1}, travels steadily at this speed and is then brought to rest with uniform deceleration. B moves with constant speed for 6 km and is then brought to rest with uniform deceleration.

(a) Sketch a velocity-time graph for the motion of the motorcycles.
(b) Find the distance A travels at constant speed.
(c) Find the initial speed of B.
(d) Find the time for which B is travelling at constant speed.

148. A ball was projected vertically upwards and it returned to its point of projection 8 seconds later.

Calculate: (i) the speed with which the ball was projected
(ii) the height to which the ball rose
(iii) the time for which the ball was above 20 m.

149. A particle travels in a straight line. Its displacement from a fixed point O, on the line, t seconds after leaving O is $5 + 3t^2 - \frac{1}{3}t^3$

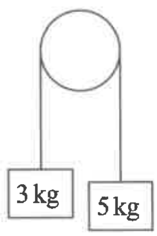
(i) Write down an expression, in terms of t, for the velocity and acceleration of the particle.

(ii) Find the maximum velocity of the particle.

150. A mass of 10 kg is at rest on a rough horizontal plane. A constant force acts on the body for 6 seconds producing an acceleration of $2\ \text{ms}^{-2}$. If the resistance to motion between the mass and the plane is $\frac{10g}{3}$ Newtons (where g is the gravitational acceleration). Calculate:

(i) the force (ii) the distance travelled after 6 seconds (iii) the velocity of the mass after 6 seconds.

151. The following diagram shows two freely hanging masses, connected by a light inextensible string, passing over a smooth fixed pulley. The system is released from rest. Find the magnitude of the tension in the string and the acceleration of the masses. If the 5 kg mass is 2 m above the floor, find the velocity that the mass hits the floor, assuming the string is long enough so that the 3 kg mass does not hit the pulley.



ANSWERS

Question 1 - 6 One required from each advantage and disadvantage, but you should know the others.

Page3

1. Advantages:
   - The elements chosen are random and free from bias.
   - Easy to use.
   - Each element of the population has an equal and known chance of selection

   Disadvantage:
   - Not suitable for large sample size.
2. Advantages:
   - Simple to use.
   - It is suitable for large sample sizes.

   Disadvantage:
   - Only random if the ordered list is random

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3. Advantages:
   - More accurate statistics obtained if there are clear strata present in the population.
   - You can have separate estimates for the different strata.

   Disadvantages:
   - If the strata are not clearly defined there may be overlap in the strata.
4. Advantages:
   - Time and cost efficient.
   - Easy to use this method.
   - Larger sample size can be used as all samples are easily accessible - being in a cluster.

   Disadvantages:
   - Each cluster may have similar elements and so the statistics obtained may be biased.
   - Since groups of elements are chosen, the researcher leaves much of the population unsampled. The diversity in the population may not be reflected in the statistics obtained.
5. Advantages:
   - Data gathering is easy as there are no restrictions on how the data is collected.
   - Administration is easy.
   - Costs are low.

   Disadvantages:
   - Not random.
   - The interviewer has to choose the members of the quota - may be bias in their choice.
6. Advantages:
   - Data from larger sample sizes can be collected easily, quickly and cheaply.

   Disadvantages:
   - Hard to justify inferences for the population.
   - Weakest form of sampling - biased.
   - Not random - interviewer has control over the members selected.

Page5

7.(a) Categorical (b) Discrete (c) Continuous (d) Continuous (e) Discrete

8. 120 + 152 + 128 = 400

Year 8 $\frac{120}{400}\times100=30$

Year 9 $\frac{152}{400}\times100=38$

Year 10 $\frac{128}{400}\times100=32$

Page6

9. $\bar{x}=\frac{3+5+7+9}{4}=6$, $s=\sqrt{5}=2.24$ to 3 s.f.

Using n – 1 divisor - OCR MEI Students
$\bar{x}=6$, $s=2.58$ to 3 s.f.

10. $\bar{x}=\frac{\sum x}{n}=\frac{184}{20}=9.2$

$s=\sqrt{47.76}=6.91$

Using n – 1 divisor - OCR MEI Students
$\bar{x}=9.2$, $s=7.09$ to 3 s.f.

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11. (i) $\sum h=9\times12=108$,

$4=\frac{\sum h^2}{n}-144$

$\sum h^2=148\times9=1332$

(ii) $\bar{x}=\frac{108+2}{10}=11$

var = $(1332+2^2)/10-11^2=12.6$

$s=\sqrt{12.6}=3.55$

Using n – 1 divisor - OCR MEI Students

(i) $\sum h=9\times12=108$,

$4=\frac{\sum h^2}{n}-144$

$\sum h^2=8\times4+108^2/9=1328$

(ii) $\bar{x}=\frac{108+2}{10}=11$

$S_{xx}=(1382+2^2)-10\times11^2=122$

$s=3.68$ to 3 s.f.

12. Remember: If a part result is corrected to 3 sig. fig. use the full calculator value if used in further working.

(a) $\bar{x}=6.78$ to 3 sig.fig.
$s=1.09857=1.10$ to 3 sig.fig.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=6.78$ to 3 sig.fig., $s=1.27$ to 3 sig.fig.

(b) $\bar{x}=33.7$
$s=20.5526=20.6$ to 3 sig.fig.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=33.7$, $s=21.7$ to 3 sig.fig.

(c) $\bar{x}=730$ to 3 sig.fig.
$s=173.767=174$ to 3 sig.fig.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=730.4$ to 3 sig.fig., $s=194$ to 3 sig.fig.

13. Edexcel students
$\bar{y}=1.3$ $S_{yy}=3625-(65^2/50)=3540.5$
$s_y=\sqrt{(3540.5/50)}=8.414868$
$\bar{x}=3\times1.3+100=103.9$
$s_x=3\times8.414868=25.2$ to 3 s.f.

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14. $\sum f=50$ $\sum fx=273$ $\sum fx^2=1715$
$\bar{x}=5.46$, $s=2.12$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=5.46$, $s=2.14$ to 3 s.f.

15. (a) $\sum f=53$ $\sum fx=436$ $\sum fx^2=3726$
$\bar{x}=8.23$ to 3 s.f. $s=1.62$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=8.23$ to 3 s.f. $s=1.64$ to 3 s.f.

(b) $\sum f=49$ $\sum fx=64$ $\sum fx^2=130$
$\bar{x}=1.31$ to 3 s.f. $s=0.973$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=1.31$ to 3 s.f. $s=0.983$ to 3 s.f.

(c) $\sum f=40$ $\sum fx=132$ $\sum fx^2=668$
$\bar{x}=3.3$ $s=2.41$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=3.3$ $s=2.44$ to 3 s.f.

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16. $\bar{x}=4$ $s=1.08$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=4$ $s=1.10$ to 3 s.f.

17. $\sum f=34$ $\sum fx=365$ $\sum fx^2=3959$
$\bar{x}=10.7$ to 3 s.f. $s=1.09$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=10.7$ to 3 s.f. $s=1.11$ to 3 s.f.

18. $\sum f=86$ $\sum fx=2197$ $\sum fx^2=64531.5$
$\bar{x}=25.5$ to 3 s.f. $s=9.89$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=25.5$ to 3 s.f. $s=9.94$ to 3 s.f.

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19. $\sum f=80$ $\sum fx=2790$ $\sum fx^2=104590$
$\bar{x}=34.9$ to 3 s.f. $s=9.55$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=34.9$ to 3 s.f. $s=9.61$ to 3 s.f.

20. $\bar{x}=1.67$ to 3 s.f. $s=0.0866$ to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x}=1.67$ to 3 s.f. $s=0.0877$ to 3 s.f.

21. Median at 50^{th} value
Median, $Q_2=\frac{8}{30}\times1.0+49.5=52.2$ cm
lower quartile at 25^{th} value
lower quartile, $Q_1=8/25\times10+39.5=42.7$ cm
upper quartile at 75^{th} value
upper quartile, $Q_3=3/18\times10+59.5=61.2$ cm
40^{th} percentile $=23/25\times10+39.5=48.7$ cm

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22. Median at 55^{th} value
Median $=\frac{17}{19}\times3+7.5=10.2$ min to 3 s.f.
lower quartile at 27.5 value
lower quartile $=6.5/17\times2+5.5=6.2647$
upper quartile at 82.5 value
upper quartile $=25.5/26\times5+10.5=15.4038$
IQR $=15.4038-6.2647=9.14$ min to 3 s.f.

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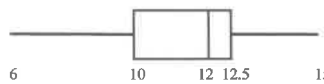
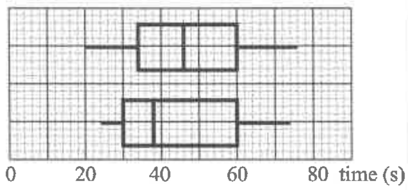
23. Median $Q_2=12$ $Q_1=10$ $Q_3=12.5$



Negative skew.

24. (a) 30 seconds Lower quartile.

(b)



(c) Any four from the following:
- Median of School B is greater than median of School A.
- Upper quartiles are the same.
- IQR of School B is less than IQR of School A.
- School B symmetrical, School A positive skew.
- Range of School B greater than range of School A.

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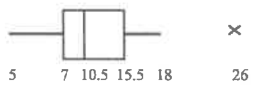
25. Median 10.5
L.q. = 7 U.q. = 15.5 IQR = 15.5 – 7 = 8.5



Almost symmetrical/slight positive skew.

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26. 8-10 group is 7.5-10.5 width 3
1.5 cm wide so 0.5 cm per min
$1.5\times3=4.5$ cm^2 represents 18 so 4 per cm^2
4 teenagers per cm^2
16-25 group Width is $10\times0.5=5$cm
$5\times$ ht = 15/4
ht = 15/20 = 0.75cm

27. For example:
1) Generally the 2^{nd} sample had longer fish.
2) 2^{nd} sample has fewer fish from 0 to 30cm.

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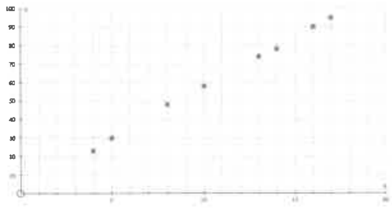
28. Answers are approximate
Median = 53
Lower Quartile = 43
Upper Quartile = 61
IQR = 61 – 43 = 18

age16

29. (a)



(b) The data appears linear. There are a fixed number of points.

(c) Approximate regression line $y = 3 + 5.5x$

(d) The slope gives an estimate of the increase in production costs for one extra lorry-load. The intercept gives an estimate of the production costs when no lorry-loads leave the factory: that is when no goods have been produced, so these costs are just overhead costs.

(e) Approximate answers
$y = 3 + 5.5 \times 12 = 69$ So costs are estimated to be 69 × £100. Say about £6900.

(f) No - extrapolation is not reliable. 30 is outside the range of given values.

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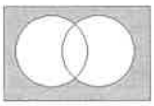
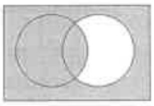
30. (a) A strong positive correlation between the two variables.
(b) A strong negative correlation between the two variables.
(c) A very weak positive correlation between the two variables. Certainly not worth considering as significant.
(d) A very weak negative correlation between the two variables. Certainly not worth considering as significant.
(e) A fair positive correlation between the two variables.
(f) A fair negative correlation between the two variables.

31. (a)



(b)



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32. (i) $P(R) = 10/20 = {}^1/_2$
(ii) $P(R \cup S) = 13/20$
(iii) $P(R' \cup S') = 17/20$

33. (i) $P(\text{it is in } X' \cap Y) = 26/80$
(ii) $P(\text{it is in } X' \cap Y') = 29/80$

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34. 11/30
35. 0.7
36. 1/5

Page20

37. 1. (i) P(Yr12) = 100/200 = ½
(ii) $P(Yr13 \cap W) = 64/200 = {}^8/_{25}$
(iii) $P(C \mid Yr13) = 8/100 = {}^2/_{25}$
$P(W \cup C) = (58+64+14+8)/200 = {}^{144}/_{200}$
P(T) = 100/200 = ½
$P((W \cup C) \cap T) = (58 + 14)/200 = {}^{72}/_{200}$
$P(W \cup C) \times P(T) = {}^{144}/_{200} \times ½$
$= {}^{72}/_{200} = P((W \cup C) \cap T)$
$\therefore$ W∪C and T are independent

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38. P(no sixes) = $1 - (5/6)^3 = 91/216 = 0.421$ to 3 s.f.

39. (i) 1/36 + 1/36 + 1/36 = 3/36 = 1/12
(ii) P(9and12) + P(10and11) +…..
= 1/36 × 1/36 + 1/36 × 1/36+…
= 10/1296 = 5/648
(iii) 1/6×1/6+1/6×1/6+…..+1/36×1/36+1/36×1/36..
= 5/36 + 6/1296 = 31/216

40. 0.8 + 0.5 – 0.9 = 0.4

41. (i) $P(X \cap Y) = P(X) \times P(Y)$ as independent
= 0.25 × 0.15 = 0.0375
(ii) $P(X \cup Y) = 0.25 + 0.15 - 0.0375 = 0.3625$

42. (a) 6/36 = 1/6 (b) 4/30 = 2/15
(c) (a) 7, 6/36 = 1/6 (b) 7, 6/30 = 1/5

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43.

0	1	2	3
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

44.

s	1	2	3	4
$P(S = s)$	0.1	0.2	0.3	0.4

45.

x	1	2	3	4	5
$P(X = x)$	4/50	7/50	10/50	13/50	16/50

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46. (a) $P(X < 3) = 0.6$
(b) $P(X \le 4) = 0.9$
(c) $P(X = 2) = F(2) - F(1) = 0.6 - 0.4 = 0.2$
(d)

1	2	3	4	5
0.4	0.2	0.1	0.2	0.1

Page24 Answers to 4 dec.pl.

47. P(three sixes from 5 throws)= ${}^5C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2$

P(four sixes from 5 throws) = ${}^5C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^1$

P(five sixes from 5 throws) = ${}^5C_5 \left(\frac{1}{6}\right)^5 = \left(\frac{1}{6}\right)^5$

48. 1. (c) $P(X = 0) = (4/5)^6 = 0.262144$
$P(X = 1) = 0.393216$
$P(X = 2) = {}^6C_2(1/5)^2(4/5)^4 = 15 \times 0.04 \times 0.4096$
$= 0.24576$
$P(X \ge 3) = 1 - [0.262144+0.393216+0.24576]$
$= 1 - 0.90112 = 0.09888 = 0.0989$

Page25 Answers to 4 dec.pl.

49. X = score out of ten
X ~ B(10,1/4)
(a) $P(X = 0) = (3/4)^{10} = 0.0563$
(b) $P(X = 1) = {}^{10}C_1(¼)^1(¾)^9 = 0.1877$
(c) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$
$P(X = 2) = {}^{10}C_2(¼)^2(¾)^8 = 0.28157$
$P(X < 3) = 0.0563+0.1877+0.28157 = 0.5256$
(d) $P(X \ge 5) = 1 - P(X<5)$
$P(X = 4) = {}^{10}C_4(¼)^4(¾)^6 = 0.145998$
$P(X = 3) = {}^{10}C_3(¼)^3(¾)^7 = 0.250282$
$P(X \ge 5) = 1 - (0.5256+0.250282+0.145998)$
$= 0.0781$

50. X = number of twos
X ~ B(8,1/3)
(a) $P(X = 6) = {}^8C_6(1/3)^6(2/3)^2 = 0.0170705685$
(b) $P(X = 7) = {}^8C_7(1/3)^7(2/3)^1 = 0.00243865264$
$P(X = 8) = (1/3)^8 = 0.00015241579$
$P(X > 5) = 0.01966 = 0.0197$

Page26 Answers to 4 dec.pl.

51. $P(X \le 6) = 0.9629$
$P(X \le 7) = 0.9887$
$P(X = 7) = 0.0258$ Calc. gives 0.0259
$P(X = 4) = 0.7687 - 0.5665 = 0.2022$
$P(X < 3) = P(X \le 2) = 0.3287$
$P(X > 8) = 1 - 0.9972 = 0.0028$

52. $P(Y \le 10) = 0.0480$
$P(Y \le 17) = 0.9645$
$P(Y = 12) = 0.2277 - 0.1133 = 0.1144$
$P(Y = 18) = 0.9924 - 0.9645 = 0.0279$ calc gives 0.0278
$P(Y < 16) = P(Y \le 15) = 0.7625$
$P(Y > 16) = 1 - P(Y \le 16) = 1 - 0.8929 = 0.1071$
$P(Y \ge 14) = 1 - P(Y \le 13) = 1 - 0.3920 = 0.6080$

Page27 Answers to 4 dec.pl.

53. (a) X = the number of red sweets
X ~ B(20, 2/3)
$P(X \ge 15) = 1 - P(Y \le 14) = 1 - 0.7028 = 0.2972$
(b) Y = the number of packets containing at least 15 red sweets
Y ~ B(5, 0.2972)
$P(Y \ge 3) = 1 - P(Y \le 2)$
$= 1 - 0.840605 = 0.1594$

54. (i) 0.3154 (ii) 0.2322

55. (i) $6 \times 0.225^5 0.775^1 = 0.00268 = 0.0027$
(ii) $0.00268 + 0.225^6 = 0.002811 = 0.0028$

56. (i) 0.7759 (ii) 0.2503

57. $P(X = 0) = (5/8)^6 = 0.0596046447$
$P(X = 1) = 6 \times (3/8)^1 \times (5/8)^5 = 0.2145767212$
(i) $P(X < 2) = P(X = 0) + P(X = 1) = 0.2742$
(ii) $P(X = 2) = 15 \times (3/8)^2 \times (5/8)^4 = 0.3218650818$
$P(X \le 2) = 0.321865 + 0.27418 = 0.5960$

58. $P(X \ge 6) = 1 - P(X \le 5) = 1 - 0.2703 = 0.7297$

59. By trial and error n =11

Page28 Answers to 4 dec.pl.

60. $P(X \ge 4) = 1 - P(X \le 3) = 1 - 0.6563 = 0.34375$
$= 0.3438$

61. $P(X \ge 3) = 1 - P(X \le 2)$
$= 1 - 0.9645 = 0.0355$

62. (i) $P(X = 3) = 0.0574$ (ii) $P(X \le 2) = 0.9298$

63. (i) $P(X = 4) = 0.2322$ (ii) $P(X \le 2) = 0.3154$
(iii) $P(X > 6) = 1 - P(X \le 6)$
$= 1 - 0.9915 = 0.0085$

64. (i) $P(X < 2) = P(X \le 1) = 0.1756$
(ii) $P(X = 5) = 0.1028$
(iii) $P(X < 5) = P(X \le 4) = 0.8298$
(iv) $P(X > 5) = 1 - P(X \le 5) = 1 - 0.932692$
$= 0.0673$
(v) $P(X \ge 5) = 1 - P(X \le 4)$
$= 1 - 0.8298468 = 0.1702$

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65. H_0: p = 0.3
H_1: p < 0.3
X ~ B (20, 0.3) 5% significance level one tailed test
$P(X \le 3) = 0.1071$
Comment: 0.1071 > 0.05 Accept H_0. No change

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66. H_0: p = 0.4
H_1: p > 0.4
X = The number of wins in 8 games
X ~ B (8, 0.4) 5% sig level one tailed test
$P(X \ge 6) = 1 - P(X \le 5) = 1 - 0.9502 = 0.0498$
$P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.8263 = 0.1737$
0.0498 < 0.05
Reject H_0, Accept H_1 he has improved his game.

67. H_0: p = 0.6
H_1: p < 0.6
X = the number that buy Bestco brand
X ~ B (12, 0.6) 10% sig level one tailed test
$P(X \le 5) = 0.1582$
$P(X \le 4) = 0.0573$
0.1582 > 0.1 Accept H_0 The evidence does not support his claim that the store sells less than expected.
Critical value:4

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68. H_0: p = 0.5
H_1: p ≠ 0.5
X = number who would vote
X ~ B (20, 0.5) 5% significance level
two tailed test ∴ 2.5% each end of distribution.
$P(X \ge 15) = 1 - P(X \le 14) = 1 - 0.9793$
$= 0.0207$
$P(X \ge 14) = 1 - P(X \le 13) = 1 - 0.9423$
$= 0.0577$
$P(X \le 5) = 0.0207$ $P(X \le 6) = 0.0577$
0.0207 < 0.025 reject H_0 acceptH_1 i.e. there is a change.
Critical region: {0,1,2,3,4,5,15,16,18,19,20}

69. H_0: p = 0.7
H_1: p ≠ 0.7
X = number of red sweets
X ~ B (20, 0.7) 8% sig level two tailed test 4% each end
$P(X \le 10) = 0.0480$ i.e. > 4%
$P(X \le 9) = 0.0171$ i.e. < 4%
Accept H_0 there is no change
$P(X \ge 18) = 1 - P(X \le 17) = 1 - 0.9645$
$= 0.0355$ i.e.< 4%
$P(X \ge 17) = 1 - P(X \le 16) = 1 - 0.8929$
$= 0.1071$ i.e. > 4%
0,1,2,3,4,5,6,7,8,9,17,18,19,20 sweets would indicate a change.

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70. Indicates maximum value/median/minimum value/upper quartile/lower quartile.
Indicates outliers.
Illustrates skewness (shape of distribution).
Allows comparisons between different data sets.
Indicates spread/range/IQR.

71. (i) Positive skew
(ii) l.q. $Q_1 = 32$, median $Q_2 = 40$, u.q. $Q_3 = 52$
(iii) $\sum x = 1286$ $\sum x^2 = 62324$
$\bar{x} = 42.9$ to 3 s.f. s = 15.5 to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x} = 42.9$ to 3 s.f. s = 15.8 to 3 s.f.

72.

Length	Frequency
$0 \le x < 20$	30
$20 \le x < 25$	36
$25 \le x < 30$	54
$30 \le x < 40$	30

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73. Median $14.5 + 10.5/16 \times 5 = 17.78125$
$= 17.8$ min
$Q_1 = 14.5 + 3/16 \times 5 = 15.4375$
$Q_3 = 19.5 + 2/8 \times 5 = 20.75$
IQR = 5.3125 = 5.31 min

74. (a) Total probability = 1
$\frac{1}{4} + k + k + \frac{1}{4} + 2k = 1$
$4k = \frac{1}{2} \quad \therefore \quad k = \frac{1}{8}$
(b) $F(2) = \frac{1}{4} + \frac{1}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$

75. Mean = 25.65min s = 9.52min to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\bar{x} = 25.65$min s = 9.77min to 3 s.f.

76.

s	0	1	2	3
$P(S = s)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

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77. $\sum m = 24 \times 28 = 672$ $\sum m^2 = 19680$
New $\sum m = 678$ $\sum m^2 = 20076$
$\bar{x} = 28.25$ s = 6.20 to 3 s.f.
Using n – 1 divisor - OCR MEI Students
$\sum m = 24 \times 28 = 672$ $\sum m^2 = 19644$
$\bar{x} = 28.25$ s = 6.21 to 3 s.f.

78. (a) P(miss,hit) + P(hit,miss) + P(miss,miss)
= 0.2 + 0.2 + 0.15 = 0.55
(b) P(hit,hit) + P(miss,hit) = 0.45 + 0.2 = 0.65

79. (i) 6 + 18 + 28 + 40 = 92
at 400g, $8/22 \times 200 = 72.72 = 73$
i.e. 400 + 73 = 473g
(ii) $\bar{x} = 100200/200 = 501$
(iii) s = 249 to 3 s.f.
Using n – 1 divisor - OCR MEI Students
s = 250 to 3 s.f.

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80. (i) Discrete data, negatively skewed.
(ii) mode 6, median 5
(iii). $\sum f = 60$ $\sum fx = 303$ $\sum fx^2 = 1827$
$\bar{x} = 5.05$ s = 2.22 to 3 s.f.
Using n – 1 divisor - OCR MEI Students
- AQA students (you are told that it is a sample.)
$\bar{x} = 10.7$ s = 2.24 to 3 s.f.

81. (a) X = Number of seeds that germinate
$X \sim B(10, 0.75)$
(i) $P(X = 8) = {}^{10}C_8 (0.75)^8 (0.25)^2 = 0.2816$
(ii) $P(X \ge 8) = 0.5256$
(b) $(0.5256)^6 = 0.0211$
(c) $X \sim B(20, 0.75)$
$H_0 = 0.75$
$H_1 > 0.75$
one tailed test at 5% level(0.05)
$P(X \ge 17) = 1 - P(X \le 16) = 1 - 0.7748 = 0.2252$
$0.2252 > 0.05$ therefore accept H_0 Not enough evidence.
$P(X \ge 18) = 1 - P(X \le 17)$
$= 1 - 0.9087 = 0.0913$ i.e. > 0.05
$P(X \ge 19) = 1 - P(X \le 18)$
$= 1 - 0.0.9757 = 0.0243$ i.e. < 0.05
Therefore the critical value is 19

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82. Assumptions:
Light, inextensible rope.
Smooth pulley.
The boat can be modelled as a particle.
The boat, chute, rope and pulley are in the same vertical plane.

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83. Total distance travelled = 500 m
Total displacement = 100 m East
Average speed = 500/180
$= 2.78$ m s^{-1} to 3 s.f.
Average velocity = 100/180
$= 0.556$ m s^{-1}East to 3 s.f.

84. Distance travelled
from A to B = 10
Displacement of B from A = + 2
Speed of body at A = 5 m s^{-1}
Velocity of body at A = + 5 m s^{-1}
Speed of body at B = 3 m s^{-1}
Velocity of body at B = – 3 m s^{-1}

85. 40000mh^{-1} = 40000/3600
= 11.1 m s^{-1} to 3 s.f.

86. $4 \times 3600 = 14400$ m h^{-1}
= 14400/1000 = 14.4 km h^{-1}

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87. (i) time = 6/12 = 0.5 hours
(ii) time = 8/20 = 0.4 hours
(iii) Av speed = 14/0.9
= 15.6 km h^{-1} to 3 s.f.

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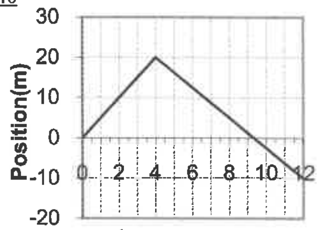
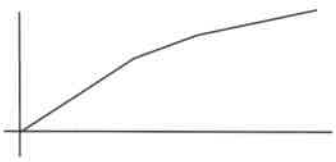
88.

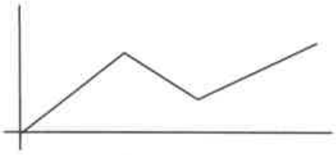


20/4 = +5 m s^{-1} – 30/8 = – 3.75 m s^{-1}

89. (i)
Sketch



(ii) First part of motion
Speed = 50/10 = 5 ms^{-1}
Second part of motion
Speed = 28/7 = 4 ms^{-1}
Last part of motion
Speed = 36/12 = 3 ms^{-1}
(iii) Average speed = 114/29
= 3.93 ms^{-1}
(iv)
sketch



(v) First part of motion
velocity = 50/10 = + 5 ms^{-1}
Second part of motion
velocity = –28/7 = – 4 ms^{-1}
Last part of motion
velocity = 36/12 = +3 ms^{-1}
(vi) Average velocity = +58/29
= +2 ms^{-1} i.e. forward

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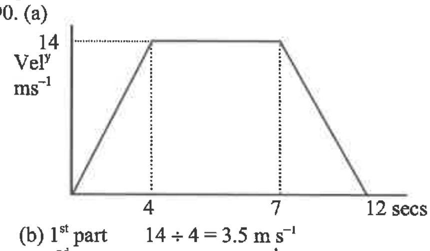
90. (a)



(b) 1st part $14 \div 4 = 3.5$ m s^{-1}
2nd part 0 m s^{-1}
3rd part $-14 \div 5 = -2.8$ m s^{-1}
Distance = $14(3 + 12) \div 2 = 105$ m

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91. u = 2 m s^{-1} v = 7 m s^{-1} $a = 1$ m s^{-2}
distance = $(2 + 7)5/2 + 3 \times 7 = 22.5 + 21$
= 43.5 m

92. (i) u = 3 m s^{-1} (ii) v = – 1.5 m s^{-1}
(iii) $a = -4.5 \div 6 = -0.75$ m s^{-2}
(iv) displacement = $12 + .5 \times 4 \times 3 - 0.5 \times 2 \times 1.5$
= 16.5 m in the +ve dir^n
(v) distance = $12 + 0.5 \times 4 \times 3 + 0.5 \times 2 \times 1.5$
= 19.5 m

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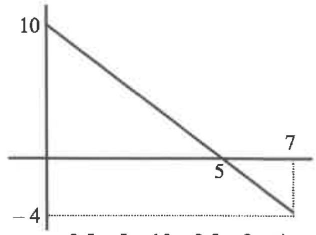
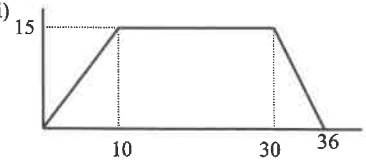
93.



Displacement = $0.5 \times 5 \times 10 - 0.5 \times 2 \times 4$
= 21 m in the +ve dir^n
Distance travelled = $0.5 \times 5 \times 10 + 0.5 \times 2 \times 4$
= 29 m

94. (i)



(ii) a = 15/10 = 1.5 m s^{-2}
(iii) decel^n = 15/6 = 2.5 m s^{-2}
(iv) $0.5 \times 10 \times 15 + 20 \times 15 + 0.5 \times 6 \times 15$
= 420 m

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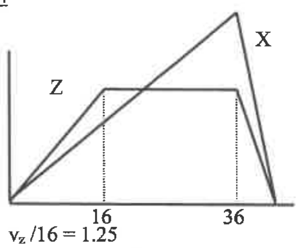
95.



(i) $v_z/16 = 1.25$
$v_z = 16 \times 1.25 = 20$ m s^{-1}
(ii) $20/t = 2$ t = 10 total time = 46
Distance = $(46 + 20)20/2 = 660$ m
(iii) t = 34.5
(iv) $660 = 46 \times v_x / 2$
$v_x = 28.7$ m s^{-1} to 3 s.f.

96. (a) (i) into (ii) $s = \frac{1}{2} t(u + u + at)$
$= \frac{1}{2} t(2u + at)$
$= ut + \frac{1}{2} at^2$
(b) From (i) $u = v - at$
into (ii) $s = \frac{1}{2} t(v - at + v)$
$= \frac{1}{2} t(2v - at)$
$= vt - \frac{1}{2} at^2$
(c) From (i) $t = (v - u)/a$
into (ii) $s = \frac{1}{2}(v - u)/a\ (u + v)$
$2as = (v - u)(u + v)$
$2as = v^2 - u^2$
$v^2 = u^2 + 2as$

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97. (a) $s = 0 + \frac{1}{2} \times 7 \times 64 = 224$
(b) $v = 5 + 3 \times 5 = 20$
(c) $s = 0 + \frac{1}{2} \times 2 \times 25 = 25$
(d) $s = \frac{1}{2}\, 10(-5 + 7) = 10$
(e) $81 = 49 + 4a$ $a = 8$
(f) $32 = u + 18$ $u = 14$
(g) $169 = 289 + 20a$ $a = -6$
(h) $32 = 4u + 16$ $u = 4$
(i) $144 = 6u - 36$ $u = 30, v = 30 - 2 \times 6 = 18$
(j) $s = 50 - 150 = -100$
(k) $-2 = 6 + 6a$ $a = -4/3$

98. '$v^2 = u^2 + 2as$'
$225 = 0 + 60a$ $a = 3.75$ m s^{-2}

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99. '$v = u + at$' gives $0 = 20 + 6a$
$\therefore a = -10/3 = -3.33$ m s^{-2} to 3 s.f.

100. $v = 96 \times 1000/3600 = 26.6667$ m s^{-1}
'$v = u + at$'
$26.6667 = 12a$ $\therefore$ a = 2.22 m s^{-2} to 3 s.f.
'$s = ut + \frac{1}{2} at^2$'
$s = 1.111111 \times 144 = 160$ m

101. '$s = ut + \frac{1}{2} at^2$' hare: $s = 5t + 0$
greyhound: $s = 0 + \frac{1}{2}\, 4(t - 1)^2$
$5t = \frac{1}{2}\, 4(t - 1)^2$
$0 = 2t^2 - 9t + 2$
t = 0.234 (not possible) or 4.27 s to 3 s.f.
s = 21.3 m

102. (i) 'v = u + at'
$4 = 1.5 + 10a \quad \therefore\ a = 0.25$ m s^{-2}
(ii) '$v^2 = u^2 + 2as$'
$= 1.5^2 + 2 \times 0.25 \times 200$
$= 102.25 \quad \therefore\ v = 10.1$ m s^{-1} to 3 s.f.

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103. (i) t = 1.53 secs (ii) $0 = 15t - 4.9t^2$
t = 3.06 secs to 3 s.f.
(iii) t = 3.14 secs to 3 s.f.
(iv) v = 15.8 m s^{-1} to 3 s.f.

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104. $4.9T^2 = 15T - 15 + 4.9T^2 - 9.8T + 4.9$
$0 = 5.2T - 10.1 \quad T = 1.942$
$s = 4.9T^2 = 18.5$ m to 3 s.f.

105. $v^2 = 0 + 39.2 \quad \therefore\ v = 6.26$ m s^{-1} to 3 s.f.

106. '$s = ut + \frac{1}{2}at^2$'
$70 = 4.9t^2 \quad t = 3.78$ secs to 3 s.f.

107. '$v^2 = u^2 + 2as$'
$= 0.25 + 490 \quad v = 22.1$ m s^{-1} to 3 s.f.
'$s = ut + ½at^2$'
$25 = 0.5t + 4.9t^2$
$0 = 4.9t^2 + 0.5t - 25$
t = 2.21 secs to 3 s.f.
or 'v = u + at'
$22.141 = 0.5 + 9.8t \quad \therefore\ t = 2.21$ secs to 3 s.f.

108. (i) '$s = ut + \frac{1}{2}at^2$'
$10 = 20t - 4.9t^2$
$4.9t^2 - 20t + 10 = 0$
t = 3.49825 or 0.58338
time above 10 m is 2.91 secs to 3 s.f.
(ii) '$s = ut + ½at^2$'
$8 = 20t - 4.9t^2$
$4.9t^2 - 20t + 8 = 0$
t = 3.632129689 or 0.44950
time above 10 m is 3.18 m

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109. v = 10t – 4
a = 10

110. $v = 5/2\ t^2 - 3t + c$
v = 5 when t = 0 ∴ 5 = c
$v = 5/2\ t^2 - 3t + 5$
$s = 5/6\ t^3 - 3/2\ t^2 + 5t + c$
s = 0 when t = 0 ∴ 0 = c
$s = 5/6\ t^3 - 3/2\ t^2 + 5t$

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111. (i) a = 4 – 2t m s^{-2}
(ii) 0 = 4 – 2t at t = 2
and v = 8 – 4 = 4 ms^{-1}
(iii) $x = 2t^2 - t^3/3 + c$
$x = 0$ when t = 0 ∴ c = 0
$x = 2t^2 - t^3/3$
t = 3, $s = 18 - 9 = 9$ m
(iv) $x = 0$ when $2t^2 - t^3/3 = 0$
$t^2(6 - t) = 0$ multiplying by 3 and factorising
t = 0 (at start) or t = 6 seconds

112. (i) $v = 20t - 3t^2 + c$
v = 7 when t = 0 ∴ c = 7
$v = 20t - 3t^2 + 7$
(ii) t = 7, v = 140 – 147 + 7 = 0
a = 20 – 6t = 0 at t = 10/3 secs.
v = 121/3 m s^{-1}
(iii) $s = \left[10t^2 - t^3 + 7t\right]_0^7 = 196$ m

113. $v = \int a\, dt$
$= at + c$
$t = 0, v = u \quad \therefore\ u = 0 + c \quad \therefore\ c = u$
$\therefore\ v = u + at$
$s = \int u + at\, dt$
$= ut + ½at^2 + c$
$t = 0, s = 0 \quad \therefore\ 0 = 0 + 0 + c \quad \therefore\ c = 0$
$\therefore\ s = ut + ½at^2$

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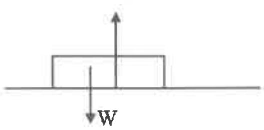
114.



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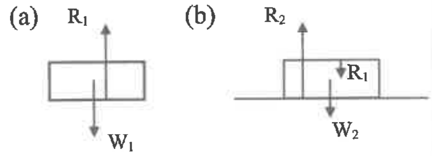
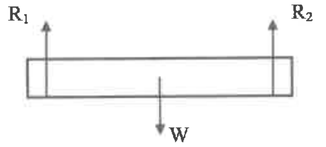
116.



117.



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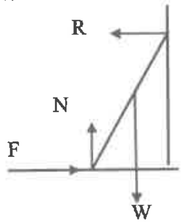
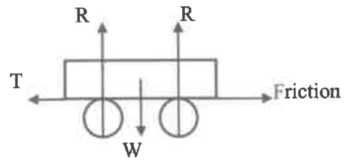
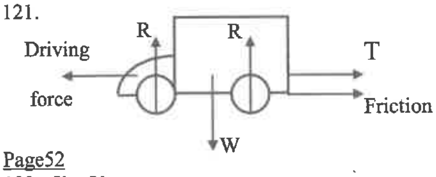
119.



120.



121.



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122. 5**i** – 2**j**

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123. (a) 29 N leftwards (b) 95 N downwards
124. 18 N downwards
125. 22 N in the 43 N direction.

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126. A = 41 N
127. X = 1690 N
128. X = 54 N Y = 62 N

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129. Vertically:
R = 20g = 20 × 9.8 = 196 N
Horizontally:
F = 140 N
130. R = 45g = 441 N
131. **i** : 2 + 5 – 2 + p = 0 p = –5
j : 4 – q + 5 – 6 = 0 q = 3
132.(i) $x = -7, y = 4$ (ii) $x = 5, y = 5$ (iii) $x = 1, y = 2$

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133. a = 25/6 = 4.17 m s^{-2} to 3 s.f.
134. F = 850 × 0.5 = 425 N
135. 'v = u + at'
8 = 2 + 3a a = 2
F = 750 × 2 = 1500 N
136. '$v^2 = u^2 + 2as$'
0 = 144 + 10a ∴ a = –14.4
F = 550 × (–14.4) = – 7920 N
Braking force is 7920 N
137. In the direction of motion:
550 – 375 = 400 × a ∴ a = 0.4375 m s^{-2}
138. 61.5 – 9.8m = 2.5 m
m = 5 kg
139. 25grams = 0.025 kg
In direction of motion:
–1000 = 0.025 a
a = – 40000
Deceleration 40 000 m s^{-2}

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140. 3**i** + 7**j** = 3***a*** ***a*** = **i** + 7/3 **j**
|**a**| = √(58/9) = 2.54 m s^{-2} to 3 s.f.
141. F = 5(5**i** – 2**j**) = 25**i** – 10**j**
Mag: $=\sqrt{(25^2+10^2)} = \sqrt{725} = 26.9$ N to 3 s.f.
tanθ = 10/25 = 2/5 θ = 21.8° to 1 d.p.
21.8° below **i** direction
or – 21.8° to the **i** direction

142. Vert: R = 3g = 29.4 N
Hor: P + 17 – 24 = 3 × 1.5
P = 11.5 N
143. Perpendicular to surface
R = 8g
Parallel to surface:
74 – 6g = 8*a*
a = 1.9 m s^{-2}

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144. For the 35kg sack:
35g – T = 35a
For the 25kg sack:
T – 25g = 25a
Add 10g = 60a
a = g/6 = 1.63 m s^{-2} to 3 s.f.
substitute T – 245 = 25 g/6
T = 286 N to 3 s.f.
145. A: T = 3a
B: 2g – T = 2a
Add 2g = 5a
a = 2g/5 = 3.92 m s^{-2}
Substitute T = 3×3.92 = 11.8 N to 3 s.f.
Pulley is smooth.
A and B are particles.
String is light and inextensible.
146. Whole system:
200 – 40 – 60 = 12a
100 = 12a
a = 8.33 m s^{-2} to 3 s.f.
block B: T – 40 = 5× 100/12
T = 81.7 N to 3 s.f.

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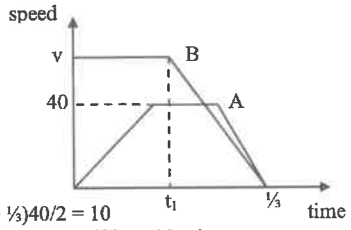
147. (a)



(b) (t + ⅓)40/2 = 10
t = 1/6 hr = 10 min.
(c) v × t_1 = 6
(t_1 + ⅓)v/2 = 10
(6/v + ⅓)v = 20
6 + v/3 = 20
18 + v = 60
v = 42 km h^{-1}
(d) t_1 = 6/42 = 1/7 hr = 8.57 minutes to 3 s.f.
148. (i) s = ut + ½ at^2
0 = 8u – 4.9 × 64
u = 39.2 ms^{-1}
(ii) t = 4, s = ut + ½ at^2
= 39.2 × 4 – 4.9 × 16 = 78.4 m
(iii) s = ut + ½ at^2
$20 = 39.2t - 4.9t^2$
$4.9t^2 - 39.2t + 20 = 0$
$t = \frac{39.2 \pm \sqrt{39.2^2 - 4\times4.9\times20}}{2\times4.9}$
= 7.4523 or 0.5477
Time above 20m = 7.4523 – 0.5477 = 6.90 s

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149. (i) $v = 6t - t^2$
a = 6 – 2t
(ii) Max when *a* = 0
0 = 6 – 2t
t = 3, v = 9
150. (i) F – 10g/3 = 10 × 2
F = 20 + 10g/3
= 52⅔ N (g = 9.8 ms^{-2})
(ii) s = ut + ½ at^2
= 0 + ½ × 2 × 36 = 36 m
(iii) v = u + at
= 0 + 2 × 6 = 12 ms^{-1}
151. 5kg: 5g – T = 5a 3kg: T – 3g = 3a
Adding: 2g = 8a ∴ a = g/4 ms^{-2} = 2.45 ms^{-2}
T = 3a + 3g = 3g/4 + 3g
= 15g/4 N = 36.75 N
s = 2 , u = 0 , a = g/4 ms^{-2}
'$v^2 = u^2 + 2as$'
$v^2 = 0 + 2 \times g/4 \times 2 = g$
v = √g ms^{-1} = 3.13 ms^{-1} to 3 s.f.